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LETTER TO THE EDITOR

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Correction to the Updated Edlén Equation for the Refractive Index of Air

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There has been significant correspondence following our recent revision of the Edlén equation for the refractive index of air [1] highlighting a sign error which occurred in our correction for the change from the IPTS-1948 to the ITS-90 temperature scale. In addition there has been some correspondence in connection with the manner of the application of the correction for the change in temperature scales. It has been suggested that rather than its incorporation into the terms of the dispersion equation, which we chose, it would have been better to modify the density term. If this alternative is adopted the terms in the dispersion equation revert to those given in the original Edlén equation [2].

The true correction factor arising from the change from IPTS-1948 to the ITS-90 temperature scale is 0,999 968 7, rather than our previously reported 1,000 030, giving an overall correction factor of 1,000 048 6. This results in the following revised form of the Edlén equations:

$$(n-1)_{tp} = \frac{(p/\mathrm{Pa})\,(n-1)_s}{96\,095,43} \times \frac{[1+10^{-8}\,(0,601-0,009\,72\,t_{90}/^{\circ}\mathrm{C})\,p/\mathrm{Pa}]}{(1+0,003\,661\,0\,t_{90}/^{\circ}\mathrm{C})}, \quad (1)$$

where $(n-1)_{tp}$ is the refractivity of air at a temperature t and atmospheric pressure p and $(n-1)_s$ is given by the revised dispersion equation:

$$(n-1)_s \times 10^8 = 8\,342,54 + 2\,406\,147\,[130-(\sigma/\mu\mathrm{m}^{-1})^2]^{-1} + 15\,998\,[38,9-(\sigma/\mu\mathrm{m}^{-1})^2]^{-1}. \quad (2)$$

For the difference in the refractive index of moist air, containing a partial pressure f of water vapour, and dry air at the same total pressure the following revised expression has been obtained:

$$n_{tpf} - n_{tp} = -(f/\mathrm{Pa}) \times [3,734\,5 - 0,040\,1\,(\sigma/\mu\mathrm{m}^{-1})^2] \times 10^{-10} \quad (3)$$

The 3σ uncertainty associated with these equations is 3×10^{-8} (mainly due to uncertainties in pressure, temperature and humidity measurement) and applies to ambient atmospheric conditions over the range of wavelengths 350 nm to 650 nm.

Directly measured values for the refractivity of air are compared with those derived from the original Edlén equation and our revised form in Table 1 which shows the resulting improvement. The results shown in the table were obtained at the NPL employing atmospheric sensors with direct traceability to national standards and also represent optimum measurement conditions. The table shows the atmospheric conditions under which the measurements were performed. All temperatures are expressed in terms of the ITS-90 scale and x is the measured level of carbon dioxide. The data in the table which relate to the original Edlén equation were computed on the assumption that no corrections were applied for the temperature differences between the ITS-90 and the IPTS-1948 scales. In addition, no corrections were applied to those levels of carbon dioxide which exceeded the values assumed in both the original Edlén equation and our revised form.

From the table, the mean of the measured carbon dioxide levels and of the differences between the measured and derived refractivities (from our revised equations as shown in the final column) are 512 ppm* and $+0,9 \times 10^{-8}$, respectively. Since our revision assumes a 450 ppm level of carbon dioxide, the

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*1 ppm $\equiv 1 \times 10^{-6}$.

Table 1. Comparison of refractivity determination at $\lambda = 633\,\text{nm}$ for ambient air.

t_{90}/°C	p/kPa	f/Pa	x/ppm	$10^8 \times (n-1)$				
				Measured	Calculated		Measured-calculated	
					Original Edlén	Revised Edlén	Original Edlén	Revised Edlén
19,526	102,094 8	1 065	510	27 392,3	27 385,1	27 392,3	+7,2	0
19,517	102,096 8	1 065	510	27 394,0	27 386,5	27 393,7	+7,5	+0,3
19,173	102,993	641	450	27 683,4	27 677,7	27 682,5	+5,7	+0,9
19,173	103,006	642	440	27 686,9	27 681,2	27 686,0	+5,7	+0,9
19,188	102,918 8	706	450	27 659,1	27 653,5	27 658,7	+5,6	+0,4
19,189	102,927 8	708	440	27 661,4	27 655,9	27 661,1	+5,5	+0,3
19,532	103,603 2	986	600	27 802,1	27 793,2	27 800,0	+8,9	+2,1
19,534	103,596 2	962	600	27 800,3	27 792,0	27 798,7	+8,3	+1,6
19,534	103,599 2	951	610	27 801,8	27 793,4	27 800,0	+8,4	+1,8

mean excess level of 62 ppm accounts for all of the $+0,9 \times 10^{-8}$ mean difference between the measured and calculated values of refractivity. This illustrates the excellent agreement between the measured and calculated values when the revised equation is employed using the additional correction for the excess level of carbon dioxide. The results give substantial confidence in the calculation of air refractivity using our revised equations.

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