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The Refractivity of Air

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The air density equation of Jones, Edlén's dispersion formula for standard air, and Edlén's empirically-derived expressions for the effects of Co_2 abundance and water vapor partial pressure on refractivity have been combined into a simplified equation for the refractivity of air, and estimates have been made of uncertainties in calculated refractivity. Under ambient conditions typical of metrology laboratories, the agreement between the simplified equation and Edlén's formulation is well within the uncertainty in each. The simplified equation is valid in the visible region.

Key words: Air density; index of refraction of air; refractivity of air; wavelength of light in air.

1. Introduction

In metrological applications of wavelengths of light in air, it is necessary to calculate the wavelength at ambient conditions of temperature (T), pressure (P), effective water vapor partial pressure (e'), and CO_2 abundance (x_{CO_2}), using the refractive index of air under these conditions. The relation between λ_{vac}, the vacuum wavelength, λ_{air}, the wavelength in air, and n, the refractive index of air, is $\lambda_{vac} = n\,\lambda_{air}$. Edlén [1] [1] has derived a dispersion formula for standard air (T = 288.15K, P = 101325 Pa, e' = 0, x_{CO_2} = 0.0003 by volume) and a formulation for the refractivity of ambient air, $(n-1)_{tpf}$. Edlén's formulation is in general use in metrology. Jones [2] has recently published a reformulation of the equation for the density of air and applied it to the transfer of the mass unit. It is the purpose of the present paper to combine the air density equation, Edlén's dispersion formula for standard air, and Edlén's empirically-derived expressions for the effects of CO_2 abundance and water vapor partial pressure on refractivity, and in so doing to develop a simpler formulation and to estimate uncertainties in the calculated refractivity.

The Edlén 1966 [1] dispersion formula for standard air is

$$(n-1)_s \times 10^8 = 8342.13 + 2406030\,(130-\sigma^2)^{-1} + 15997\,(38.9-\sigma^2)^{-1}, \quad (1)$$

where n is the refractive index, σ is the vacuum wave number, $(1/\lambda_{vac})$, in μm^{-1} and standard air is dry air at 288.15K, 101325 Pa and a CO_2 abundance of 0.0003 by volume. Edlén [1] expressed the refractivity, $(n-1)_{tp}$ of dry air at temperature t (in °C) and pressure p (in torr) as

$$(n-1)_{tp} = K_\lambda D_{tp}, \quad (2)$$

where K_λ [3] is a dispersion factor which is independent of t and p, and the density factor, D_{tp}, is

$$D_{tp} = p\,(1 + \epsilon_t\,p)/\left\{(1 + \alpha t)\left[1 - \frac{(n-1)_{tp}}{6}\right]\right\}, \quad (3)$$

where α = 1/273.15 and ϵ_t is a factor which multiplies p in an expression for the nonideality of the gas. By substituting suitable values, (3) becomes

$$D_{tp} = p\,[1 + p\,(0.817 - 0.0133\,t) \times 10^{-6}]/(1 + 0.0036610\,t). \quad (4)$$

For air with a CO_2 abundance of x by volume, Edlén derived

$$(n-1)_x = [1 + 0.540\,(x - 0.0003)]\,(n-1)_s, \quad (5)$$

and,

$$n_{tph} - n_{tp} = -h\,(5.722_4 - 0.0457\,\sigma^2) \times 10^{-8} \quad (6)$$

for the difference in refractive index of moist air holding h torr of water vapor at a total pressure p. (To avoid using the same symbol for two different quantities, in the present work h has been substituted for Edlén's f).

From (4) and the relation

$$(n-1)_{tp} = (n-1)_s\,D_{tp}/D_s, \quad (7)$$

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[1] Figures in brackets indicate literature references at the end of this paper

Edlén's general formula is

$$(n-1)_{tp} = \frac{p\,(n-1)_s}{720.775}\cdot \frac{[1+p\,(0.817-0.0133\,t)\times 10^{-6}]}{[1+0.0036610\,t]}, \tag{8}$$

where D_s (= 720.775) is the density factor for standard air.

Equations (5), (6), and (8) are generally combined in the calculation of the refractivity of moist air, in the visible region.

2. Present Formulation

In the following, the air density equation derived by Jones [2] will be incorporated into a refractivity equation. The density of moist air, ϱ, is given by [2]

$$\varrho = \frac{P\,M_a}{RTZ}\left[1-\left(1-\frac{18.0152}{M_a}\right)\frac{U}{100}\frac{f\,e_s}{P}\right], \tag{9}$$

where P is the pressure in Pa, M_a is the apparent molecular weight of dry air, R is the universal gas constant, T is the temperature in kelvins, Z is the compressibility factor (the non-ideality of the air-water vapor mixture is reflected in the departure of Z from 1), U is the relative humidity in percent, and f is the enhancement factor (a factor which expresses the fact that the effective saturation vapor pressure of water in air is greater than the saturation vapor pressure, e_s, of pure phase over a plane surface of pure ordinary liquid water). Tables of Z, e_s and f are provided in the appendix of the present paper.

The Lorentz-Lorenz [4,5] formulation of the Clausius-Mossotti [6,7] equation can be expressed as

$$\frac{n^2-1}{n^2+2} = C\,\frac{\varrho_a}{M_a}, \tag{10}$$

the left side of which can be approximated [1] by $\frac{2}{3}(n-1)\cdot[1-(n-1)/6]$. Therefore,

$$(n-1) = C'\,\frac{\varrho_a}{M_a}\left[1-\frac{(n-1)}{6}\right]^{-1}, \tag{11}$$

where ϱ_a and M_a are the density and apparent molecular weight, respectively, of dry air and C and C' are constants. Since $\varrho_a = PM_a/RTZ$ [2], (11) becomes

$$(n-1) = \frac{C'P}{RTZ\left[1-\frac{(n-1)}{6}\right]}, \tag{12}$$

and for standard air,

$$(n-1)_s = \frac{C'P_s}{RT_sZ_s\left[1-\frac{(n-1)_s}{6}\right]}. \tag{13}$$

By dividing (12) by (13),

$$(n-1) = \frac{\left[1-\frac{(n-1)_s}{6}\right]}{\left[1-\frac{(n-1)}{6}\right]}\,\frac{P/TZ}{P_s/T_sZ_s}\,(n-1)_s. \tag{14}$$

By substituting the appropriate values of P_s (101325 P_a), T_s (288.15K) and Z_s (0.99958 from table 1 in the appendix), (14) becomes

$$(n-1) = 0.0028426\,\frac{P}{TZ}\,\frac{\left[1-\frac{(n-1)_s}{6}\right]}{\left[1-\frac{(n-1)}{6}\right]}\,(n-1)_s, \tag{15}$$

which, when rearranged, becomes

$$(n-1)^2 - 6(n-1) + 0.0170556\,(n-1)_s\cdot\left[1-\frac{(n-1)_s}{6}\right]\frac{P}{TZ} = 0. \tag{16}$$

The appropriate square root of (16) is

$$(n-1) = 3 - \left\{9 - 0.0028426\,(n-1)_s\cdot[6-(n-1)_s]\,\frac{P}{TZ}\right\}^{1/2}. \tag{17}$$

We shall return now to Edlén's development and combine (2) with (3):

$$(n-1)_{tp} = K_\lambda\,D_{tp} = \frac{K_\lambda p\,(1+\epsilon_t p)}{(1+\alpha t)\left[1-\frac{(n-1)_{tp}}{6}\right]}. \tag{18}$$

$(1+\epsilon_t p)$ is recognized to be $1/Z$, $(1+\alpha t) = T/273.15$, and $p = 760\,P/101325$; therefore,

$$(n-1)_{tp} = \frac{760\times 273.15}{101325}\,\frac{K_\lambda P}{TZ}\,\frac{1}{\left[1-\frac{(n-1)_{tp}}{6}\right]}. \tag{19}$$

By comparing (19) with (12), $K_\lambda R(760\times 273.15)/101325$ is seen to correspond to C'.

It remains now to combine (17) with Edlén's empirically-derived expressions for the effects of CO_2 abundance, (5), and water vapor partial pressure, (6), to arrive at the general expression:

$$(n-1) = 3 - \left\{9 - (n-1)_x\,[6-(n-1)_x]\cdot 0.0028426\,\frac{P}{TZ}\right\}^{1/2} - f e_s\,\frac{U}{100}\,(0.042922 - 0.000343\,\sigma^2)\times 10^{-8}, \tag{20}$$

where e_s is in Pa. Equation (20) corresponds to (8) combined with (5) and (6), i.e. Edlén's formulation [1]. The agreement between the refractivity of moist air calculated using (20) and Edlén's formulation is illustrated for $T = 293.15\text{K}$, $P = 101325$ Pa, $U = 50$, $x_{CO_2} = 0.00043$, $Z = 0.99963$ (from table 1), $f = 1.0041$ (from table 2), $e_s = 2338$ Pa (from table 3) and $\lambda_k = \sigma^{-1} = 0.6329912714$ μm for an iodine stabilized helium-neon laser [8]. Using (20), $(n - 1) = 27131._0 \times 10^{-8}$; using Edlén's formulation $(n - 1)_{tph} = 27131._3 \times 10^{-8}$. For a more extreme case ($T = 288.15\text{K}$, $P = 70000$ Pa, $U = 50$, $x_{CO_2} = 0.00080$, $Z = 0.99971$, $f = 1.0030$, $e_s = 1705$ Pa, (for the same wavelength), (20) gives $(n - 1) = 19069._6 \times 10^{-8}$, and the Edlen formulation gives $(n - 1)_{tph} = 19068._1 \times 10^{-8}$. As will be demonstrated in the next section, the difference between the results for the two formulations is well within the uncertainty of each.

Equation (15) can be approximated by

$$(n-1) = 0.0028426 \frac{P}{TZ} (n-1)_s; \qquad (21)$$

in the first of the above examples, the resulting change is 0.02×10^{-8} which is negligible. Equation (20) then becomes

$$(n-1)_{TPe}' = 0.0028426 \frac{P}{TZ} (n-1)_x - f e_s \frac{U}{100}(0.042922 - 0.000343\, \sigma^2) \times 10^{-8}, \qquad (22)$$

where the subscript TPe' follows Edlén's convention, $e' = fe_sU/100$. For a CO_2 abundance of 0.0003 by volume and a vacuum wavelength of 0.6329912714 μm (22) becomes

$$(n-1)_{TPe}' = (78.603 \frac{P}{TZ} - 0.042066 f e_s \frac{U}{100}) \times 10^{-8}. \qquad (23)$$

The variation of CO_2 abundance, x, can be incorporated in (23) by multiplying 78.603 by $[1 + 0.540\,(x - 0.0003)]$. At NBS, a constant value of 1.0042 can be used for f [2] with negligible effect on calculated $(n-1)_{TPe'}$. Equation (23) then becomes

$$(n-1)_{TPe}' = (78.603 \frac{P}{TZ} - 0.042243\, e_s \frac{U}{100}) \times 10^{-8}. \qquad (24)$$

3. Estimation of Uncertainties

We follow the suggested practice of Eisenhart [9, 10] in stating separately the random and systematic components of the estimated uncertainties. The stated random component is one standard deviation; the stated systematic component is one-third of the half-width of the interval between the bounds on the systematic error.

The uncertainties in calculated $(n-1)_{TPe'}$ due to estimated uncertainties [2] in P, T, Z, U, f, e_s, and x can be estimated from equation (22). We shall not attempt to estimate the uncertainties in Edlén's [1] dispersion formula for standard air and his expressions for the effects of CO_2 abundance and water vapor partial pressure. The state-of-the-art in pressure measurement [11] permits the measurement of pressure in a laboratory with a random relative uncertainty of less than $\pm$ 0.02 percent, calibration of pressure measuring instruments against a primary standard of pressure contributes a systematic relative uncertainty of about $\pm$ 0.003 percent. The corresponding uncertainties in $(n-1)_{TPe'}$, in the first example above are $\pm 5.4 \times 10^{-8}$ and $\pm 0.8 \times 10^{-8}$.

The measurement of temperature in the air path is potentially as critical as the pressure measurement, in terms of its effect on the uncertainty in the calculated $(n-1)_{TPe'}$; it is possible to make only a rough estimate of the uncertainty in the temperature measurement. If the vicinity of the path were instrumented with a network of thermopile junctions, the measurements would be expected to have a standard deviation of about $\pm$ 0.05K [12] and a systematic uncertainty of the order the $\pm$ 0.01K. The corresponding uncertainties in $(n-1)_{TPe'}$ in the first example are $\pm 4.6 \times 10^{-8}$ and $\pm 0.9 \times 10^{-8}$.

The estimated systematic relative uncertainty in the compressibility factor, Z, for the first example is $\pm$ 0.0017 percent. The corresponding uncertainty in $(n-1)_{TPe'}$ is $\pm 0.5 \times 10^{-8}$.

The uncertainty in calculated $(n-1)_{TPe'}$ due to humidity measurement can be estimated from the second term in (22). The state-of-the-art in humidity measurement [13] permits the measurement of relative humidity, U, with a random uncertainty of $\pm$ 0.5 percent relative humidity and a systematic uncertainty of $\pm$ 0.3 percent relative humidity. The corresponding uncertainties in $(n-1)_{TPe'}$ in the first example are $\pm 0.5 \times 10^{-8}$ and $\pm 0.3 \times 10^{-8}$. The uncertainties contributed by uncertainties in f and e_s are negligible [2].

The uncertainty in calculated $(n-1)_{TPe'}$ due to a variation in CO_2 abundance, x, can be estimated from (5). In the first example, a variation in x of $\pm$ 0.0001 corresponds to a systematic uncertainty in $(n-1)_{TPe'}$ of $\pm 1.5 \times 10^{-8}$.

The overall random uncertainty in $(n-1)_{TPe'}$, estimated by combining the random uncertainties by quadrature, is $\pm 7.1 \times 10^{-8}$. The overall systematic uncertainty, estimated by combining the addition, is $\pm 2.5 \times 10^{-8}$. The systematic uncertainty due to variation in CO_2 abundance is necessarily not included. It should be emphasized that these uncertainties are based on the *best possible measurements* of P, T and U.

4. Direct Determination of Air Density

In 1967, Bowman and Schoonover [14] used a pair of stainless steel weights (one of which was hollow) of nearly equal mass but of grossly different volume to make direct determination of the air density in a balance case, thus avoiding the uncertainties in the parameters and environmental variables in air density calculations. A similar scheme will be used in the transfer of the mass unit [15].

Having estimated the uncertainty in calculated $(n-1)_{TPe'}$ due to the uncertainties in the various variables to be about $\pm 1 \times 10^{-7}$ at the level of the equivalent of 1 standard deviation, it is of interest to estimate how much improvement would result from the *direct* determination of air density, ϱ, if practicable. From (9),

$$\frac{P}{TZ} = \frac{\varrho R}{M_a} \frac{1}{\left[1 - (1 - \frac{18.0152}{M_a}) \frac{U}{100} \frac{f e_s}{P}\right]}, \quad (25)$$

where $M_a = 28.963 + 12.011\,(x_{CO_2} - 0.00033)$; recalling that ϱ is the density of moist air. By substituting (25) in (22),

$$(n-1)_{TPe'} = 0.0028426 \frac{\varrho R}{M_a} \cdot \frac{(n-1)_x}{\left[1 - (1 - \frac{18.0152}{M_a}) \frac{U}{100} \frac{f e_s}{P}\right]} - f e_s \frac{U}{100} (0.042922 - 0.000343\,\sigma^2) \times 10^{-8}. \quad (26)$$

The uncertainties in the various parameters in (25), other than ϱ and $(n-1)_x$, are taken from [2]. The resulting overall uncertainty in the calculated $(n-1)_{TPe'}$ are $\pm 1.9 \times 10^{-8}$ random and $\pm 1.8 \times 10^{-8}$ systematic. The uncertainty due to the effect on M_a of a variation of x_{CO_2}, 1.1×10^{-8} per 0.0001, has necessarily not been included. It can be concluded that even it the uncertainty in a direct determination of ϱ were negligible, the uncertainty in $(n-1)_{TPe'}$ due to the uncertainties in the various variables and parameters would be reduced by a factor of about 2.5. The major contributors to the uncertainty in $(n - 1)_{TPe'}$ are the uncertainties in R, M_a and U.

5. Conclusions

Jones's air density equation [2], Edlén's [1] dispersion formula for standard air, and Edlén's empirically-derived expressions for the effects of CO_2 abundance and water vapor partial pressure on refractivity have been combined into a simple refractivity of air equation, and estimates have been made of uncertainties in calculated refractivity.

The general equation is (22), which is valid in the visible region; tables of Z, f and e_s have been included in the appendix of this paper. The overall estimated uncertainty is about $\pm 1 \times 10^{-7}$ at the level of the equivalent of 1 standard deviation. The major contributors to the uncertainty in refractivity are the uncertainties in the measurements of pressure and temperature. The magnitude of the uncertainty due to variation in CO_2 concentration can approach that of the uncertainties due to the pressure and temperature measurements. Therefore, the CO_2 concentration should be treated as a variable and should be observed.

If it were practicable to make a direct measurement of air density representative of the air path, the uncertainty in calculated refractivity due to the uncertainties in the various variables and parameters would be reduced by a factor of about 2.5.

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6. References

[1] Edlén, B. The refractive index of air. Metrologia. 2: 71-80; 1966.

[2] Jones, F. E. The air density equation and the transfer of the mass unit. J. Res. Nat. Bur. Stand. (U.S.). 83(5): 419-428; 1978 September-October.

[3] Barrell, H.; Sears, J. E., Jr. The refraction and dispersion of air for the visible spectrum. Phil. Trans. Roy. Soc. London. A238: 1-64; 1939.

[4] Lorentz, H. A. Ueber die Bezienhung zwischen der Fortpflanzungsgeschwindigkeit des Lichtes and der Korperdichte. Ann. Physik u. Chemie. N. F. 9: 641-665; 1880.

[5] Lorenz, L. Ueber die Refractionsconstante. Ann. Physik u. Chemie. N. F. 11: 70-103; 1880.

[6] Clausius, R. Die Mechanische Warmetheorie, Vol. 2. Braunschweig: Friedrich Vieweg und Sohn; 1879. 62-97.

[7] Mossotti, O. F. Memorie di matematica e di fisica della Societa Italiana delle Scienze Residente in Modena. (26)(2): 49-74; 1850.

[8] Schweitzer, W. G.; Kessler, E. G., Jr.; Deslattes, R. D.; Layer, H. P.; Whetstone, J. R. Description, performance, and wavelengths of iodine stabilized lasers. Appl. Opt. 12(12): 2927-2938; 1973.

[9] Eisenhart, C. Expression of the uncertainties of final results. Science. 160: 1201-1204; 1968.

[10] Eisenhart, C. Realistic evaluation of the precision and auccuracy of instrument calibration systems. J. Res. Nat. Bur. Stand. (U.S.). 67C: 161-187; 1963 April-June.

[11] Heydemann, P. L. M. Private communication.

[12] Roeser, W. F.; Wensel, H. T. *Temperature, its measurement and control in science and industry.* New York: Reinhold Publ. Corp.; 1941. p. 311.

[13] Wexler, A. Measurement of humidity in the free atmosphere near the surface of the earth. Meteorological Monographs. 11(33): 262-282; 1970.

[14] Bowman, H. A.; Schoonover, R. M.; Jones, M. W. Procedure for high precision density determinations by hydrostatic weighing. J. Res. Nat. Bur. Stand. (U.S.). 71C(3): 179-198; 1967 July-September.

[15] Schoonover, R. M. Private communication.

[16] Hyland, R. W. A correlation of the second virial coefficients and enhancement factors for moist air. J. Res. Nat. Bur. Stand. (U.S.). 79A(4): 551-560; 1975 July-August.

[17] Wexler, A.; Greenspan, L. Vapor pressure equation for water in the range 0 to 100 °C. J. Res. Nat. Bur. Stand. (U.S.). 75A(3): 213-230; 1971 May-June.

7. Appendix

TABLE 1. Compressibility factor, Z, for air containing reasonable amounts of CO_2 [2]

Temperature (Celsius)	Pressure (pascals)	(mm Hg)	Relative Humidity in Percent 0	25	50	75	100
15.0	70000	525.0	.99971	.99970	.99968	.99967	.99965
	75000	562.5	.99969	.99968	.99966	.99965	.99963
	80000	600.0	.99966	.99966	.99964	.99963	.99961
	85000	637.6	.99964	.99963	.99962	.99961	.99959
	90000	675.1	.99962	.99961	.99960	.99959	.99957
	95000	712.6	.99960	.99959	.99958	.99957	.99955
	100000	750.1	.99958	.99957	.99956	.99955	.99953
	101325	760.0	.99958	.99957	.99956	.99954	.99953
	105000	787.6	.99956	.99955	.99954	.99953	.99951
	110000	825.1	.99954	.99953	.99952	.99951	.99949
16.0	70000	525.0	.99971	.99970	.99969	.99967	.99965
	75000	562.5	.99969	.99968	.99967	.99965	.99963
	80000	600.0	.99967	.99966	.99965	.99963	.99962
	85000	637.6	.99965	.99964	.99963	.99961	.99960
	90000	675.1	.99963	.99962	.99961	.99959	.99958
	95000	712.6	.99961	.99960	.99959	.99958	.99956
	100000	750.1	.99959	.99958	.99957	.99956	.99954
	101325	760.0	.99959	.99958	.99956	.99955	.99953
	105000	787.6	.99957	.99956	.99955	.99954	.99952
	110000	825.1	.99955	.99954	.99953	.99952	.99950
17.0	70000	525.0	.99972	.99971	.99970	.99968	.99966
	75000	562.5	.99970	.99969	.99968	.99966	.99964
	80000	600.0	.99968	.99967	.99966	.99964	.99962
	85000	637.6	.99966	.99965	.99964	.99962	.99960
	90000	675.1	.99964	.99963	.99962	.99960	.99958
	95000	712.6	.99962	.99961	.99960	.99958	.99956
	100000	750.1	.99960	.99959	.99958	.99956	.99954
	101325	760.0	.99960	.99959	.99957	.99956	.99954
	105000	787.6	.99958	.99957	.99956	.99954	.99953
	110000	825.1	.99956	.99955	.99954	.99952	.99951
18.0	70000	525.0	.99973	.99972	.99970	.99968	.99966
	75000	562.5	.99971	.99970	.99968	.99966	.99964
	80000	600.0	.99969	.99968	.99966	.99964	.99962
	85000	637.6	.99967	.99966	.99964	.99963	.99960
	90000	675.1	.99965	.99964	.99962	.99961	.99959
	95000	712.6	.99963	.99962	.99961	.99959	.99957
	100000	750.1	.99961	.99960	.99959	.99957	.99955
	101325	760.0	.99961	.99960	.99958	.99957	.99955
	105000	787.6	.99959	.99958	.99957	.99955	.99953
	110000	825.1	.99957	.99956	.99955	.99953	.99951
19.0	70000	525.0	.99973	.99972	.99971	.99968	.99966
	75000	562.5	.99972	.99970	.99969	.99967	.99964
	80000	600.0	.99970	.99968	.99967	.99965	.99963
	85000	637.6	.99968	.99967	.99965	.99963	.99961
	90000	675.1	.99966	.99965	.99963	.99961	.99959
	95000	712.6	.99964	.99963	.99961	.99960	.99957
	100000	750.1	.99962	.99961	.99959	.99958	.99956
	101325	760.0	.99962	.99960	.99959	.99957	.99955
	105000	787.6	.99960	.99959	.99958	.99956	.99954
	110000	825.1	.99958	.99957	.99956	.99954	.99952
20.0	70000	525.0	.99974	.99973	.99971	.99969	.99966
	75000	562.5	.99972	.99971	.99969	.99967	.99964
	80000	600.0	.99970	.99969	.99967	.99965	.99963
	85000	637.6	.99969	.99967	.99966	.99964	.99961
	90000	675.1	.99967	.99966	.99964	.99962	.99959
	95000	712.6	.99965	.99964	.99962	.99960	.99958
	100000	750.1	.99963	.99962	.99960	.99958	.99956
	101325	760.0	.99963	.99961	.99960	.99958	.99956
	105000	787.6	.99961	.99960	.99958	.99957	.99954
	110000	825.1	.99959	.99958	.99957	.99955	.99953
21.0	70000	525.0	.99975	.99973	.99971	.99969	.99966
	75000	562.5	.99973	.99972	.99970	.99967	.99964
	80000	600.0	.99971	.99970	.99968	.99966	.99963
	85000	637.6	.99969	.99968	.99966	.99964	.99961
	90000	675.1	.99968	.99966	.99965	.99962	.99960
	95000	712.6	.99966	.99965	.99963	.99961	.99958
	100000	750.1	.99964	.99963	.99961	.99959	.99956
	101325	760.0	.99964	.99962	.99961	.99959	.99956
	105000	787.6	.99962	.99961	.99959	.99957	.99955
	110000	825.1	.99960	.99959	.99958	.99956	.99953
22.0	70000	525.0	.99975	.99974	.99972	.99969	.99966
	75000	562.5	.99974	.99972	.99970	.99968	.99964
	80000	600.0	.99972	.99971	.99969	.99966	.99963
	85000	637.6	.99970	.99969	.99967	.99964	.99961
	90000	675.1	.99968	.99967	.99965	.99963	.99960
	95000	712.6	.99967	.99965	.99963	.99961	.99958
	100000	750.1	.99965	.99964	.99962	.99960	.99957
	101325	760.0	.99965	.99963	.99961	.99959	.99956
	105000	787.6	.99963	.99962	.99960	.99958	.99955
	110000	825.1	.99962	.99960	.99958	.99956	.99954
23.0	70000	525.0	.99976	.99975	.99972	.99969	.99966
	75000	562.5	.99974	.99973	.99971	.99968	.99964
	80000	600.0	.99973	.99971	.99969	.99966	.99963
	85000	637.6	.99971	.99969	.99967	.99965	.99962
	90000	675.1	.99969	.99968	.99966	.99963	.99960
	95000	712.6	.99968	.99966	.99964	.99962	.99959
	100000	750.1	.99966	.99964	.99962	.99960	.99957
	101325	760.0	.99965	.99964	.99962	.99960	.99957
	105000	787.6	.99964	.99963	.99961	.99958	.99956
	110000	825.1	.99963	.99961	.99959	.99957	.99954
24.0	70000	525.0	.99977	.99975	.99973	.99969	.99965
	75000	562.5	.99975	.99973	.99971	.99968	.99964
	80000	600.0	.99973	.99972	.99970	.99967	.99963
	85000	637.6	.99972	.99970	.99968	.99965	.99962
	90000	675.1	.99970	.99969	.99966	.99964	.99960
	95000	712.6	.99968	.99967	.99965	.99962	.99959
	100000	750.1	.99967	.99965	.99963	.99961	.99957
	101325	760.0	.99966	.99965	.99963	.99960	.99957
	105000	787.6	.99965	.99964	.99962	.99959	.99956
	110000	825.1	.99964	.99962	.99960	.99957	.99954
25.0	70000	525.0	.99977	.99976	.99973	.99970	.99965
	75000	562.5	.99976	.99974	.99971	.99968	.99964
	80000	600.0	.99974	.99972	.99970	.99967	.99963
	85000	637.6	.99973	.99971	.99968	.99965	.99962
	90000	675.1	.99971	.99969	.99967	.99964	.99960
	95000	712.6	.99969	.99968	.99965	.99962	.99959
	100000	750.1	.99968	.99966	.99964	.99961	.99958
	101325	760.0	.99967	.99966	.99963	.99961	.99957
	105000	787.6	.99966	.99964	.99962	.99960	.99956
	110000	825.1	.99965	.99963	.99961	.99958	.99955
26.0	70000	525.5	.99978	.99976	.99973	.99970	.99965
	75000	562.5	.99976	.99975	.99972	.99968	.99964
	80000	600.0	.99975	.99973	.99970	.99967	.99963
	85000	637.6	.99973	.99971	.99969	.99966	.99961
	90000	675.1	.99972	.99970	.99967	.99964	.99960
	95000	712.6	.99970	.99968	.99966	.99963	.99959
	100000	750.1	.99969	.99967	.99964	.99961	.99958
	101325	760.0	.99968	.99966	.99964	.99961	.99957
	105000	787.6	.99967	.99965	.99963	.99960	.99956
	110000	825.1	.99966	.99964	.99961	.99959	.99955
27.0	70000	525.0	.99979	.99977	.99974	.99969	.99964
	75000	562.5	.99977	.99975	.99972	.99968	.99963
	80000	600.0	.99976	.99974	.99971	.99967	.99962
	85000	637.6	.99974	.99972	.99969	.99966	.99961
	90000	675.1	.99973	.99971	.99968	.99964	.99960
	95000	712.6	.99971	.99969	.99966	.99963	.99959
	100000	750.1	.99970	.99968	.99965	.99962	.99958
	101325	760.0	.99969	.99967	.99965	.99961	.99957
	105000	787.6	.99968	.99966	.99964	.99960	.99956
	110000	825.1	.99966	.99965	.99962	.99959	.99955
28.0	70000	525.0	.99979	.99977	.99974	.99969	.99964
	75000	652.5	.99978	.99976	.99972	.99968	.99963
	80000	600.0	.99976	.99974	.99971	.99967	.99962
	85000	637.6	.99975	.99973	.99970	.99966	.99961
	90000	675.1	.99973	.99971	.99968	.99965	.99960
	95000	712.6	.99972	.99970	.99967	.99963	.99959
	100000	750.1	.99970	.99968	.99966	.99962	.99958
	101325	760.0	.99970	.99968	.99965	.99962	.99957
	105000	787.6	.99969	.99967	.99964	.99961	.99956
	110000	825.1	.99967	.99965	.99963	.99959	.99955

TABLE 2. Values of enhancement factor, f, calculated [2] from Hyland's data [16]

Pressure, pascals	t, C 15	20	25	30
70 000	1.0030	1.0031	1.0032	1.0034
75 000	1.0032	1.0033	1.0034	1.0035
80 000	1.0033	1.0034	1.0035	1.0037
85 000	1.0035	1.0036	1.0037	1.0038
90 000	1.0036	1.0037	1.0038	1.0040
95 000	1.0038	1.0039	1.0040	1.0041
100 000	1.0039	1.0040	1.0042	1.0043
101 325	1.0040	1.0041	1.0042	1.0043
105 000	1.0041	1.0042	1.0043	1.0045
110 000	1.0043	1.0043	1.0045	1.0046

TABLE 3. Values of saturation water vapor pressure, e_s, calculated using formulation of Wexler and Greenspan [17]

e_s, pascals

Temperature, C

	15	16	17	18	19	20	21	22	23	24	25	26	27
.00	1705	1818	1938	2064	2197	2338	2487	2644	2810	2985	3169	3363	3567
.05	1711	1824	1944	2070	2204	2346	2495	2652	2818	2994	3178	3372	3577
.10	1716	1830	1950	2077	2211	2353	2503	2660	2827	3003	3188	3382	3588
.15	1722	1836	1956	2083	2218	2360	2510	2669	2836	3012	3197	3392	3598
.20	1727	1841	1962	2090	2225	2367	2518	2677	2844	3021	3207	3402	3609
.25	1733	1847	1968	2097	2232	2375	2526	2685	2853	3030	3216	3413	3619
.30	1738	1853	1975	2103	2239	2382	2533	2693	2861	3039	3226	3423	3630
.35	1744	1859	1981	2110	2246	2390	2541	2701	2870	3048	3235	3433	3641
.40	1749	1865	1987	2116	2253	2397	2549	2709	2879	3057	3245	3443	3651
.45	1755	1871	1994	2123	2260	2404	2557	2718	2887	3066	3255	3453	3662
.50	1761	1877	2000	2130	2267	2412	2565	2726	2896	3075	3264	3463	3673
.55	1766	1883	2006	2136	2274	2419	2573	2734	2905	3085	3274	3473	3683
.60	1772	1889	2012	2143	2281	2427	2580	2743	2914	3094	3284	3484	3694
.65	1778	1895	2019	2150	2288	2434	2588	2751	2922	3103	3294	3494	3705
.70	1783	1901	2025	2157	2295	2442	2596	2759	2931	3112	3303	3504	3716
.75	1789	1907	2032	2163	2302	2449	2604	2768	2940	3122	3313	3515	3727
.80	1795	1913	2038	2170	2310	2457	2612	2776	2949	3131	3323	3525	3738
.85	1801	1919	2044	2177	2317	2464	2620	2785	2958	3140	3333	3535	3749
.90	1806	1925	2051	2184	2324	2472	2628	2793	2967	3150	3343	3546	3759
.95	1812	1931	2057	2190	2331	2480	2636	2801	2976	3159	3353	3556	3770