SIMULATION OF MATHEMATICAL PHASE NOISE MODEL FOR A PHASE-LOCKED-LOOP

Amornchai Amornthipparat, Amnuay Rangsiwatakapong, and Duangrat Eungdamrong
Sirindhron International Institute of Technology, Thammasat University

Abstract

Phase noise in a phase-locked-loop (PLL) is originated from each electronic component in the PLL itself. The pattern of phase noise is derived from the plot of power spectrum density in frequency domain. By using a reliable phase noise model, the output phase noise due to each noise source is, therefore, predicted correctly by calculating the relation between an input power spectrum density and its closed-loop transfer function. There are four noise sources considered in PLL, which are generated by reference oscillator, voltage-controlled oscillator, filter, and main divider. This paper uses a regenerative divider in place of the main divider to improve the phase noise in phase-locked-loop. Since regenerative divider is based on feedback system, the phase noise is more satisfied. A simulation of the output phase noise is done using MATLAB.

1. Introduction

The world of communication technology seems to grow dramatically and continuously, especially in wireless communication. Mobile phones have become the basic gadgets for most people. Due to this growth, excellent performance of equipment is needed. To improve the performance of the receiving terminal of wireless communication, the effect of the phase noise on phase-locked-loop (PLL) has been studied.

Phase – locked – loop (PLL) is a feedback system used to lock the output frequency and phase to the frequency and phase of input. It can also be used as a frequency synthesizer for modulation and demodulation. Normally, PLL is used in both transmitting and receiving terminals for any wireless communication. It is composed of several components, which are phase detector, filter, voltage controlled oscillator, and main divider. It operates as a negative feedback loop. While the voltage controlled oscillator generates an output signal, its output phase is fed and compared to the reference signal. This process continues until no phase difference exists. At this state, PLL is called "frequency locked". Noise in PLL is classified into two categories, which are amplitude noise and phase noise. Amplitude noise is detected and terminated easily. In contrast, phase noise is difficult to identify and express in an equation due to unpredictable characteristics of electronic components. It is, therefore, important to study the characteristic of the phase noise because it affects the system performance and the signal to noise ratio. Moreover, the benefit of having a more reliable phase noise model deals mainly with the design problem [1].

This paper assumed that four noises appear in PLL which are originated by reference oscillator, loop filter, voltage-controlled oscillator, and main divider. To investigate the phase noise using MATLAB, noise is inserted into the circuit as an additional source. Its individual characteristic depends on the representing device shown in Figure 1.

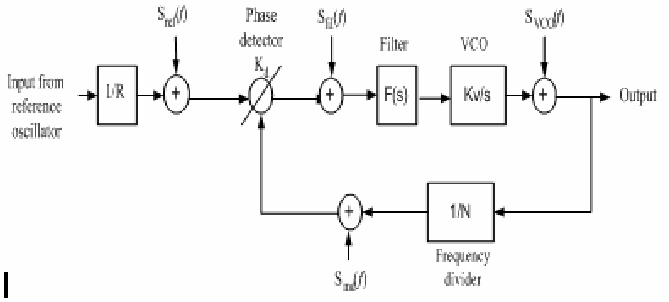

Figure 1 PLL block diagram

2. Noise model

The prototype of phase noise spectrum density is described in frequency domain as a decaying nonlinear graph, which can be classified into several regions depending on its slope. The slope deviates from 0 to -40 dB/dec or less. The graph is depicted in Figure 2.

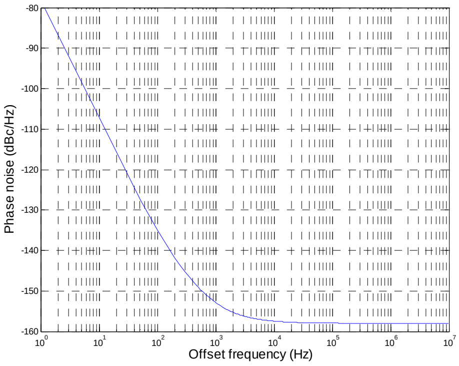

Figure 2 Prototype of phase noise spectrum density

This prototype allows us to construct an equation to represent the phase noise model. According to Eq (1), the first term (k_o) represents thermal noise which acquires 0 dB/dec slope region. The second term (k_1/f) is flicker noise which represents –10 dB/dec slope region and so on [2].

$$S_\phi(f) = k_0 + \frac{k_1}{f} + \frac{k_2}{f^2} + \frac{k_3}{f^3} + \frac{k_4}{f^4} \quad (1)$$

Each model of noise source may contain different regions of slope and its characteristic is, therefore, different.

VCO noise model:

$$S_{\phi\_VCO}(f) = k_{0\_VCO} + \frac{k_{2\_VCO}}{f^2} + \frac{k_{3\_VCO}}{f^3} \quad (2)$$

Reference oscillator noise model:

$$S_{\phi\_ref}(f) = k_{0\_ref} + \frac{k_{1\_ref}}{f} + \frac{k_{2\_ref}}{f^2} + \frac{k_{3\_ref}}{f^3} \quad (4)$$

Filter noise model:

$$S_{\phi\_filter} = k_{0\_R1} + k_{0\_op-amp} + \frac{k_{1\_op-amp}}{f} \quad (5)$$

In this paper, a second-order active low-pass filter, as shown in Figure 3, is used. The active low-pass filter amplifies the signal while filtering out the high frequency which exceeds the PLL's bandwidth. Noise generated by the resistor, R1, and op-amp are considered. Furthermore, noise from the capacitor is considerably small compared to R1.

a) Resistor noise:

Noise in resistor is due to random motion of electrons in the resistor. The noise model available for a resistor is assumed to be thermal noise and is defined by Eq 6, which shows power dissipated in the resistor. This type of noise should be taken into account since there are resistors presented in the loop filter [3].

$$P_r(R) = 4kTBR \quad (6)$$

where
P_r = Thermal noise
k = Boltzmann's constant
T = Absolute temperature (K)
B = Bandwidth of the filter (Hz)
R = Resistor (Ω)

Noise from the resistor has a spectrum density as follows:

$$S_{\phi\_R1}(f) = k_{0\_R1} = 4kTBR \quad (7)$$

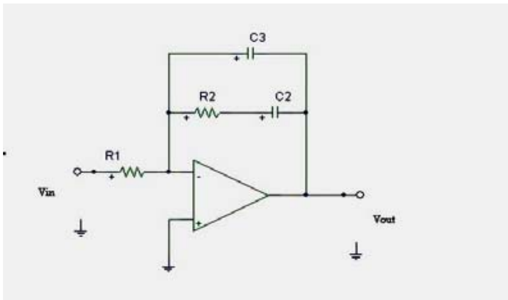

Figure 3 Circuit of second order active low pass filter

b) Operational amplifier noise:

Op-amp noise is derived by the experiment or given by the manufacturer. The Op-amp noise consists of a flicker noise and thermal. Therefore, only k_{0_op-amp}, and k_{1_op-amp} are considered for the op-amp noise.

$$S_{\phi\_op-amp}(f) = k_{0\_op-amp} + \frac{k_{1\_op-amp}}{f} \quad (7)$$

Regenerative divider noise model:

Frequency divider is an important component in phase-locked-loop, especially for frequency synthesizer. It made the phase-locked-loop to be more versatile in multi-loop phase-locked-loop. Regenerative divider is another type of analog frequency divider, which basically consists of a mixer, a filter and a multiplier. It relies on the feedback system to generate an oscillation. The mixer and filter identify the system to be either positive or negative feedback system.

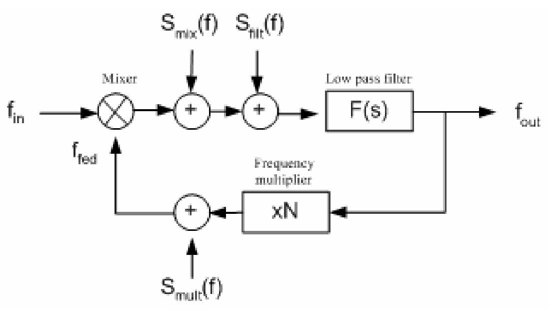

Figure 4 Regenerative divider with noise diagram

A first-order active low-pass filter is used because it provides more flexibility in adjusting the gain that affects the performance of the regenerative divider.

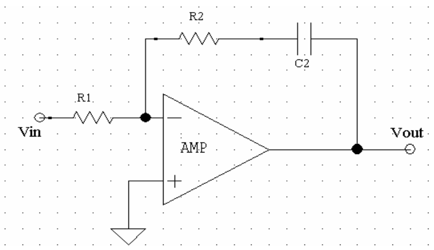

Figure 5 First-order active low-pass filter

From Figure 4, the output frequency is multiplied by N and mixed with the input. In the low-pass filter, a high frequency component is suppressed. The output frequency (f_{out}) is, therefore, equal to $f_{in} - f_{fed}$. The transfer function is described in Eq 8, [4]-[6].

$$f_{out} = \frac{1}{N+1} \cdot f_{in} \quad (8)$$

Noises in the regenerative divider are theoretically originated from the mixer, the low pass filter, and the frequency multiplier. Using a superposition theorem, noises from each component can be combined.

$$S_{md}(f) = S_{mix}(f) + S_{md\_filt}(f) + S_{mult}(f) \quad (9)$$

The output power spectrum density is calculated by multiplying the input power spectrum density with the magnitude square of closed-loop transfer function, $\theta(jf)$, which is defined as (forward loop/1+open loop). The position of the input varies depending on the investigating noise. It determines the ability of the system to terminate phase noise. By investigating the phase noise in term of spectrum density, the output of phase noise is stated as follows [7]:

$$S_{\phi\_out}(f) = S_{\phi\_source}(f) \cdot |\theta(jf)|^2 \quad (10)$$

where

$\theta(jf)$ = closed-loop transfer function of each noise source
$S_{\phi\_out}(f)$ = output phase spectrum density
$S_{\phi\_source}(f)$ = input phase spectrum density

This method is reliable because the constant (*k*) for each electronic component such as op-amp is derived from the datasheet of voltage noise density vs. frequency. Since power is directly proportional to voltage square, Eq. 11, the unit of voltage is then transformed to unit power before evaluating the coefficients, k_0, k_1.

$$P \propto V^2 \quad (11)$$

$$P(dB) \propto 10\log(V^2) \quad (12)$$

The value of k_0 denotes the noise floor of the op-amp. Its value is evaluated from the power at 1 Hz of the tangent line, with 0 dB/dec slope. The value of k_1 is calculated from the power at 1 Hz of the tangent line with -10 dB/dec slope [4]. The other component of *k* is calculated according this manner.

$$P(dB) = 10\log(k) \quad (13)$$

By taking logarithm base 10 and multiplying by 10, phase noise in dB can be obtained. In the next section, phase noise is simulated by MATLAB program using the phase noise model mentioned above.

3. Discussion

The noise modified by the closed-loop transfer function is observed by using MATLAB.

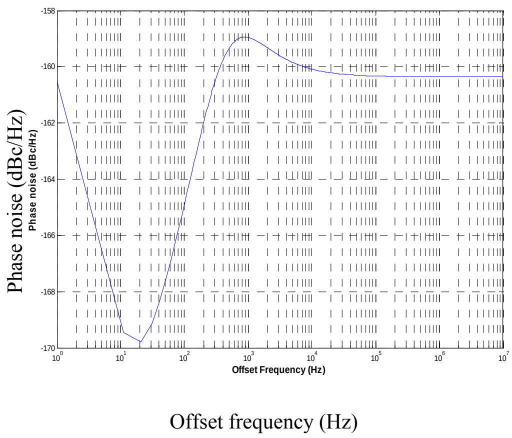

Figure 6: Regenerative divider phase noise spectrum density

Figure 6 represents the phase noise of regenerative divider using first-order low-pass filter. It starts decreasing from -161 dBc/Hz to its minimum point -169.8 dBc/Hz at 21.5 Hz and then raising with the slope of 7 dB/dec untill it reaches -159 dBc/Hz at 900 Hz. The phase noise gradually decreases to its noise floor, -160.4 dBc/Hz at 30 kHz.

The phase noise of the regenerative divider gives out a better attenuation at low offset frequencies, which is useful in practice because it reduces problem during frequency inter- modulation process.

Noise from other components modified by the loop is plotted in Figure 7. The shape of the noise is determined by the closed-loop transfer function of that particular noise source.

The closed-loop transfer function of VCO noise acts as a high pass filter, which allows the high frequency component to pass, and suppresses the rest.

Reference noise has a different type of closed-loop transfer function. It acts as a low pass filter in this case. This noise causes a significant effect on the total noise at frequencies lower than the bandwidth. For higher frequencies, the effect is negligible.

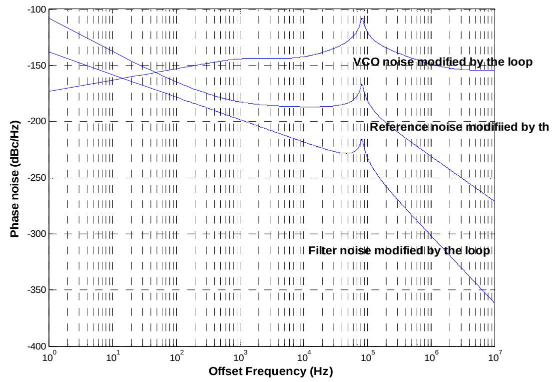

Figure 7 Phase noise of VCO, reference, and filter. $k_{0\_VCO}=10^{-15.5}$, $k_{2\_VCO}=10^{-3}$, $k_{3\_VCO}=10^{0.7}$, $k_{0\_md}=10^{-15.5}$, $k_{1\_md}=10^{-12.5}$, $k_{0\_ref}=10^{-15.8}$, $k_{1\_ref}=10^{-12.7}$, $k_{2\_ref}=10^{-9.86}$, $k_{3\_ref}=10^{-7.82}$, $K_v=10^7$, $k_{0\_R1}=10^{-15.38}$, $k_{0\_op\_amp}=10^{-17.045}$, $k_{1\_op\_amp}=10^{-16.02}$ $K_p=0.5$, $N=3$, $R_1=10$, $R_2=5.2$, $C_2=27\times10^{-6}$, $C_3=3.8\times10^{-6}$

For the filter, its closed-loop transfer function behaves as a bandpass filter, which has passband frequencies around the bandwidth. Since the filter noise modified by the loop is considerably high compared to the other, the significant effect to the output noise can, therefore, be observed above bandwidth frequency.

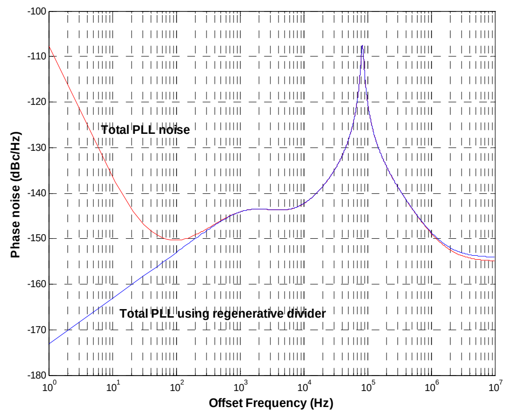

Figure 8 Compared total noise.

Figure 8 shows the comparison between total PLL noise using and not using regenerative divider. After substituting main divider by regenerative divider, phase noise is reduced significantly at the low frequency offset, while the rest is almost unchanged. At 1 Hz offset frequency, the significant decreasing of 60dBc/Hz is observed. However, a phase noise increases at high frequency offset.

The plot of total noise describes the ability of the system to achieve phase noise reduction in low offset frequency with a tiny disturbance at the high offset frequencies. The peak phase noise of -118dBc/Hz occurs at 84 kHz offset frequency. Noise floor of both cases is about -155 dBc/Hz.

PLL mostly consists of active components such as VCOs, mixers, frequency multipliers, which are vulnerable to the surroundings. Therefore, it is likely to have an unexpected result such as a peak or non-uniform phase noise.

4. Conclusion

To improve the phase noise, a regenerative frequency divider is used. It reduces the phase noise at low frequency. However, more noise sources must be considered for practical circuit in order to have an accurate phase noise model. In our model, the regenerative divider noise is taken into account along with VCO, reference oscillator and filter noise. Each noise source affects to the total output differently. VCO noise affects on the frequency higher than the bandwidth, while the lower frequency reflects the consequence of regenerative divider and reference oscillator.

Reference

[1] Jack R. Smith, ‘Modern Communication Circuits’, 2nd Edition, New York: WCB/Mc Graw-Hill, 1998

[2] Eric Drucker, ‘Model PLL Dynamics and Phase – Noise Performance’, Microwaves & RF, May 2000

[3] Devendra K. Misra, ‘Radio-Frequency and Microwave Communication circuits’, John Wiley & Sons, 2001

[4] Micheal M. Driscoll, ‘Phase Noise Performance of Analog Frequency Divider’, IEEE Transactions On Ultrasonics, Ferroelectrics, And Frequency Control. Vol.37, No. 4, July 1990.

[5] E.S. Ferre-Pikal and F.L. Walls, ‘Low PM Noise Regenerative Dividers’, 1997 IEEE International Frequency Control Symposium.

[6] http://www.eecg.toronto.edu/~kphang/ece1371/freqregen.pdf

[7] B.P. Lathi, ‘Modern Digital and Analog Communication System’, 3rd Edition, Oxford University Press, New York, 1998.