Home Search Collections Journals About Contact us My IOPscience

Higher-order effects on uncertainties of clocks of Mg atoms in an optical lattice

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2017 J. Phys.: Conf. Ser. 793 012020

(http://iopscience.iop.org/1742-6596/793/1/012020)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 192.53.103.119 This content was downloaded on 20/06/2017 at 10:41

Please note that terms and conditions apply.

You may also be interested in:

Fluorescence power of an atom near polarizable particles P Robles, F Claro and R Rojas

Higher-order effects on self-similar parabolic pulse in the microstructured fibre amplifier Liu Wei-Ci, Xu Wen-Cheng, Feng Jie et al.

Excitation Spectrum of Spin-1 Bosonic Atoms in anOptical Lattice with High Filling Factors Hou Jing-Min

Accurate values of polarizabilities from high-angular-momentum Rydberg states S H Patil

Wavefunctions for the confined hydrogen atom based oncoalescence and inflexion properties S H Patil

Hertz-Level Clock Spectroscopy of 171Yb Atoms in a One-Dimensional Optical Lattice Meng-Jiao Zhang, Hui Liu, Xi Zhang et al.

Magic Wavelengths for a Lattice Trapped Rubidium Four-Level Active Optical Clock Zang Xiao-Run, Zhang Tong-Gang and Chen Jing-Biao

Optical lattice clocks towards the redefinition of the second F Bregolin, G Milani, M Pizzocaro et al.

Static polarizability of condensed particles A Kh Kyarov and A P Savintsev

# Higher-order effects on uncertainties of clocks of Mg atoms in an optical lattice

V D Ovsiannikov<sup>1</sup>, S I Marmo<sup>1</sup>, S N Mokhnenko<sup>1</sup> and V G Palchikov<sup>2,3</sup>

<sup>1</sup> Voronezh State University, Voronezh 394006, Russia

<sup>2</sup> FGUP "VNIIFTRI", Mendeleevo 141570, Moscow Region, Russia

<sup>3</sup> National Research Nuclear University "MEPhI", Moscow 115409, Russia

E-mail: ovd@phys.vsu.ru

**Abstract.** Multipole, nonlinear and anharmonic effects on the optical-lattice-based clocks of Mg atoms are evaluated theoretically. Dipole polarizabilities, hyperpolarizabilities and multipolar polarizabilities for Mg atoms are calculated in the single-electron approximation with the use of analytical presentations for the wave and Green's functions in the modified model-potential approach. For comparison, the data are also given for atoms of the group IIb elements (Zn, Cd, Hg).

#### 1. Introduction

The recent progress in detecting experimentally the  ${}^{3}P_{0} - {}^{1}S_{0}$  clock transition in neutral Mg atoms and determination of corresponding magic wavelength (MWL)  $\lambda_{mag} = 468.46$  nm [1] paves the way to the development of a new time-frequency standard of Mg atoms in an optical lattice. To ensure the Mg clock fractional uncertainties below 10<sup>-17</sup>, the role of nonlinear and non-dipole effects of interaction between trapped atoms and the magic-frequency lattice should be evaluated.

In this paper, we present theoretical considerations of the most appropriate operational conditions based on results of numerical calculations of electromagnetic susceptibilities of Mg atoms in their clock states. The single-electron Fues' model-potential (FMP) approach was used with modifications introduced for the most efficient account of contributions from the valence electrons of the  $3s^2$  shell and from their interaction with the inner-electron shells [2, 3]. The modifications of the FMP parameters were performed, taking into account the contribution of the 3s electron to the polarizability of the upper clock state  $3s3p(^{3}P_{0})$ . The modified approach improved significantly the agreement of the calculated MWL (471.6 nm) with the above-cited result of experimental measurements.

Together with electric-dipole (E1) frequency-dependent polarizabilities  $\alpha_{e(g)}^{E1}(\omega)$  of the excited (e) and ground (g) clock states the magnetic dipole  $\alpha_{e(g)}^{M1}(\omega)$  and electric quadrupole  $\alpha_{e(g)}^{E2}(\omega)$  polarizabilities (M1 and E2) provide their specific contributions to the dynamic Stark shift. The E2-M1 contributions to the linear in the laser intensity *I* Stark shifts of the clock energy levels are added to the E1 shift in a traveling wave and subtracted in a standing wave, due to the on-phase and a quarter-period off-phase temporal and spatial distributions of corresponding interactions [4, 5].

The shift of the clock frequency in a traveling wave is determined by the difference of polarizabilities  $\Delta \alpha^{\Sigma}(\omega) = \alpha_{e}^{\Sigma}(\omega) - \alpha_{g}^{\Sigma}(\omega)$ :

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

Published under licence by IOP Publishing Ltd

$$\Delta v_{clock} = \Delta \alpha^{\Sigma}(\omega) \frac{I}{4} + \Delta \beta(\omega, \xi) \frac{I^2}{16},$$
(1)

where  $\alpha_{e(g)}^{\Sigma}(\omega) = \alpha_{e(g)}^{E1}(\omega) + \alpha_{e(g)}^{qm}(\omega)$ , and  $\alpha_{e(g)}^{qm}(\omega) = \alpha_{e(g)}^{E2}(\omega) + \alpha_{e(g)}^{M1}(\omega)$  is the sum of the E2-M1 polarizabilities, which in the far-off-resonance regions of frequencies is usually 6 to 7 orders smaller than the dipole polarizability  $\alpha_{e(g)}^{E1}(\omega)$ . The quadratic in *I* shift, determined by the difference of the clock-state hyperpolarizabilities  $\Delta\beta(\omega,\xi) = \beta_e(\omega,\xi) - \beta_g(\omega,\xi)$ , is also taken into account in (1). The second argument  $\xi$  indicates the dependence of hyperpolarizabilities on the circular polarization degree  $\xi$  [6], which for the clock states may be written as

$$\beta_{e(g)}(\omega,\xi) = \beta_{e(g)}^{l}(\omega) + \xi^{2} \Big[ \beta_{e(g)}^{c}(\omega) - \beta_{e(g)}^{l}(\omega) \Big],$$
<sup>(2)</sup>

where two independent components of the hyperpolarizability tensor determine the quadratic shift of excited (ground-state) energy level in the field of linearly polarized ( $\beta_{e(g)}^{l}(\omega)$ ) and circularly polarized ( $\beta_{e(g)}^{c}(\omega)$ ) laser radiation.

The clock-frequency shift in a field of a lattice standing wave has quite different dependence on the lattice-laser intensity. In particular, this dependence includes in addition to linear and quadratic in I terms, those proportional to  $\sqrt{I}$  and to  $I^{3/2}$ , which are related with the vibrational motion of an atom trapped in the lattice-induced dynamic Stark-effect energy well. Corresponding equations and their backgrounds are described in section 2 of this paper. In section 3 the numerical data of calculations on the basis of the model-potential approach in the single-electron approximation are presented for magnesium atoms in comparison with earlier calculated data [3] for some other atoms of the group II elements.

Atomic units  $e = m = \hbar = 1$  are used throughout the paper, unless otherwise stated explicitly. The speed of light in atomic units c = 137.036 coincides numerically with the inverse fine-structure constant  $\alpha = 1/137.036$ .

### 2. Shifts of clock levels in the field of a lattice wave

The Stark effect on atomic energy levels enables trapping neutral atoms in the minima of thus created potential-energy wells in a lattice standing wave. But the Stark energies of the ground and excited clock states cause the clock-frequency shift which should be taken into account in evaluating the clock-frequency uncertainties.

The Stark energies are determined by the interaction of a trapped atom with a lattice wave of an electric-field

$$\mathbf{E}(X,t) = 2\mathbf{E}_0 \cos(kX)\cos(\omega t), \tag{3}$$

and a magnetic field (a quarter-period off-phase, both in space and time)

$$\mathbf{B}(X,t) = 2\left[\mathbf{e}_{x} \times \mathbf{E}_{0}\right] \sin(kX) \sin(\omega t), \qquad (4)$$

oscillating in time with frequency  $\omega$  and in space along the incident laser beam of intensity  $I = cE_0^2/8\pi$  and the wave vector  $\mathbf{k} = k\mathbf{e}_x$ ,  $k = \omega/c$ ; X is a displacement of an atom from the lattice standing-wave antinode in the lattice-laser-beam direction determined by the unit vector  $\mathbf{e}_x$ . The operator of atom-lattice interaction may be presented as  $\hat{V}(X,t) = \text{Re}\{\hat{V}(X)\exp(-i\omega t)\}$ , where the spatial factor determines an X-distribution of electric dipole  $\hat{V}_{E1} = (\mathbf{r} \cdot \mathbf{E}_0)$ , electric quadrupole  $\hat{V}_{E2} = \omega r^2 (\{\mathbf{E}_0 \otimes \mathbf{n}\}_2 \cdot \mathbf{C}_2(\theta, \varphi))/c\sqrt{6}$  and magnetic dipole  $\hat{V}_{M1} = ([\mathbf{n} \times \mathbf{E}_0] \cdot (\hat{\mathbf{J}} + \hat{\mathbf{S}}))/2c$  operators [3, 7]

$$\hat{V}(X) = \hat{V}_{E1}\cos(kX) + (\hat{V}_{E2} + \hat{V}_{M1})\sin(kX).$$
(5)

Note, that the operators are written with account of doubling the amplitude of an incident laser wave  $\mathbf{E}_0$  in the amplitudes (3) and (4) of a standing wave.

With account of the second- and fourth-order in  $\hat{V}(X)$  terms of perturbation theory, linear and quadratic in the lattice-laser intensity I correspondingly, the interactions (5) produce lattice potential wells for an atom in its ground (g) or excited (e) state, which may be presented, as follows [8]

$$U_{g(e)}^{latt}(X;I,\omega,\xi) \approx -D(I,\omega,\xi) + U_{g(e)}^{harm}(I,\omega,\xi)X^2 - U_{g(e)}^{anh}(I,\omega,\xi)X^4$$
(6)

The separation of neighbour wells equals half wavelength  $\lambda$  of the lattice wave. To exclude tunneling atoms between potential wells, its location near the well bottom (at the equilibrium position X = 0) should hold the inequality  $|X| \ll \lambda/4$ , where  $\lambda/4$  is the distance between the top and bottom of the well. At this condition the ratio between absolute values of consecutive terms in the right-hand side of equation (6) is significantly smaller than the unit. Therefore, the motion of a trapped atom is described by the wavefunctions of an anharmonic oscillator. The depth of the potential well (6)

$$D_{g(e)}(I,\omega,\xi) = \alpha_{g(e)}^{E1}(\omega)I + \beta_{g(e)}(\omega,\xi)I^2,$$
(7)

is determined by the lattice-frequency-dependent electric dipole polarizability  $\alpha_{g(e)}^{E1}(\omega)$  and hyperpolarizability  $\beta_{g(e)}(\omega,\xi)$  of the ground (excited) state. At an operational intensity the quadratic term in the right-hand side of equation (7) is at least 6 orders of magnitude smaller than the linear one. The coefficient of the harmonic term, determining the frequency of vibrations  $\Omega_{e(g)}(I,\omega,\xi)$  in a lattice trap, depends on a combined E1-E2-M1 polarizability  $\alpha_{e(g)}^{dqm}(\omega) = \alpha_{e(g)}^{E1}(\omega) - \alpha_{e(g)}^{qm}(\omega)$  and on the hyperpolarizability, as follows

$$U_{g(e)}^{harm}(I,\omega,\xi) = \frac{\mathcal{M}\Omega_{g(e)}^{2}(I,\omega,\xi)}{2} = \left[\alpha_{g(e)}^{dqm}(\omega)I + 2\beta_{g(e)}(\omega,\xi)I^{2}\right]k^{2}.$$
(8)

From this equation the eigenfrequency of the atomic vibrations inside the potential well (6) may be presented as

$$\Omega_{g(e)}(I,\omega,\xi) = 2\sqrt{\mathcal{E}^{rec}\left[D_{g(e)}(I,\omega,\xi) - \alpha_{g(e)}^{qm}(\omega)I + \beta_{g(e)}(\omega,\xi)I^2\right]},$$
(9)

where  $\mathcal{E}^{rec} = k^2 / (2\mathcal{M}) = \omega^2 / (2\mathcal{M}c^2)$  is the recoil energy of a lattice photon,  $\mathcal{M} \approx 1823\mathcal{A}$  is the mass of atom in the atomic (electron mass) units,  $\mathcal{A}$  is the mass of atom in the atomic mass units (1/12 of the carbon atom  ${}^{12}C_6$  mass), presented for each atom in periodic tables. It becomes evident, that together with linear and quadratic in intensity terms, the dependence of the energy-level shifts in the field of the lattice wave includes also the terms of half-integer powers of I.

The coefficient of the anharmonic term is

$$U_{g(e)}^{anh}(I,\omega,\xi) = \left[\alpha_{g(e)}^{dqm}(\omega)I + 5\beta_{g(e)}(\omega,\xi)I^{2}\right]\frac{k^{4}}{3} = \frac{k^{2}\mathcal{M}\Omega_{g(e)}^{2}(I,\omega,\xi)}{6} + k^{4}\beta_{g(e)}(\omega,\xi)I^{2}.$$
 (10)

Thus, the atom trapped in the potential energy well (6) effectuates vibrations in a bound state of an oscillator quantum number n and of the vibration energy

$$\mathbb{E}_{g(e)}^{vib}(I,\omega,\xi,n) = -D_{g(e)}(I,\omega,\xi) + \Omega_{g(e)}(I,\omega,\xi) \left(n + \frac{1}{2}\right) - \mathbb{E}_{g(e)}^{anh}(I,\omega,\xi) \left(n^{2} + n + \frac{1}{2}\right), \quad (11)$$

added to the energy of atom in its ground (excited) state. Here the anharmonic constant is determined by the recoil energy, and the fraction of hyperpolarizability over the E1-E2-M1 polarizability  $\alpha_{e(g)}^{dqm}(\omega)$ , as follows

$$\mathbb{E}_{g(e)}^{anh}(I,\omega,\xi) = \frac{\mathcal{E}^{rec}}{2} \left[ 1 + \frac{3\beta_{g(e)}(\omega,\xi)I}{\alpha_{g(e)}^{dam}(\omega)} \right].$$
(12)

Therefore, the hyperpolarizability contributes, via the anharmonic effect, to the linear in *I* term of the vibration energy (11), which is however essentially smaller than the linear term of the depth (7), since the fraction in the brackets at the operational intensity is on the order of  $10^{-7}$ - $10^{-6}$ .

#### 3. Lattice-induced clock-frequency shift

Thus, the shift of the clock-transition frequency in a lattice-trapped atom is determined by the difference of vibration energies (11) of atom in clock states:

$$\Delta v_{cl}^{latt}(I,\omega,\xi,n) = \mathbb{E}_{e}^{vib}(I,\omega,\xi,n) - \mathbb{E}_{g}^{vib}(I,\omega,\xi,n)$$
$$= -\Delta D(I,\omega,\xi) + \Delta \Omega(I,\omega,\xi) \left(n + \frac{1}{2}\right) - \Delta \mathbb{E}^{anh}(I,\omega,\xi) \left(n^{2} + n + \frac{1}{2}\right),$$
(13)

where the oscillation quantum number n is one and the same for the excited and ground-state atom, independent of transition between its clock states (the Lamb-Dicke regime). Therefore the dependence of the clock-frequency shift for an atom in the field of the lattice standing wave differs significantly from that of the traveling wave (1) and may be presented, as follows:

$$\Delta v_{cl}^{latt}(n, I, \omega, \xi) = c_{1/2}(n, \omega)I^{1/2} + c_1(n, \omega, \xi)I + c_{3/2}(n, \omega, \xi)I^{3/2} + c_2(\omega, \xi)I^2.$$
(14)

The half-integer powers of the laser intensity I appear in the right-hand side of this equation from the eigenfrequency (9) of atomic oscillations in lattice traps. The basic contributions to this shift comes from the difference of electric-dipole polarizabilities  $\Delta \alpha^{E1}(\omega) = \alpha_e^{E1}(\omega) - \alpha_e^{E1}(\omega)$ , which should vanish at the "magic" frequency  $\omega_m$ , determined as the root of equation  $\alpha_e^{E1}(\omega_m) - \alpha_g^{E1}(\omega_m) = 0$ . Equalization of the E1 polarizabilities means the equivalence of only the linear terms of the depths (7) of the Stark-effect energy wells. However, an inevitable in practice uncertainty of the magic frequency  $\Delta \omega_m$  may introduce corresponding uncertainties into the differences of electric dipole polarizabilities, transferred into the differences  $\Delta D$  and  $\Delta \Omega$ . The latter appear in the coefficients  $c_{1/2}^{E1}(n) \equiv c_{1/2}(n,\omega_m)$  and  $c_1^{E1}(n,\xi) \equiv c_1(n,\omega_m,\xi)$  of the square-root and linear in I terms of the resolution (14) taken at the magic frequency  $\omega_m$ . The multipole (E2-M1) effects, transferred into the difference  $\Delta\Omega = \Omega_{e} - \Omega_{o}$ , appear only in the square-root term of (14), whereas effects of the hyperpolarizability appear in all three terms  $\Delta D$ ,  $\Delta \Omega$  and  $\Delta E^{anh}$  of equation (13) and finally contribute to the values of all the rest coefficients  $c_j^{E_1}(n,\xi) = c_j(n,\omega_m,\xi)$  (j=1, 3/2, 2). So, the coefficients in the right-hand side of (14) may be presented in terms of the magic frequency shift  $\Delta \omega_m$ , the differences of multipole polarizabilities  $\Delta \alpha_m^{qm} = \alpha_e^{qm}(\omega_m) - \alpha_g^{qm}(\omega_m)$  and the differences of hyperpolarizabilities  $\Delta\beta_m(\xi) = \beta_e(\omega_m, \xi) - \beta_s(\omega_m, \xi)$ , as follows:

$$c_{1/2}^{E1}(n) = \left(\frac{\partial \Delta \alpha_m^{E1}}{\partial \omega} \Delta \omega_m - \Delta \alpha_m^{qm} \left(n + \frac{1}{2}\right)\right) \sqrt{\frac{\mathcal{E}_{E1}^{rec}}{\alpha_m^{E1}}},$$

$$c_1^{E1}(n,\xi) = -\frac{\partial \Delta \alpha_m^{E1}}{\partial \omega} \Delta \omega_m - \frac{3\mathcal{E}_{E1}^{rec}}{2\alpha_m^{E1}} \Delta \beta_m(\xi) \left(n^2 + n + \frac{1}{2}\right),$$

$$c_{3/2}^{E1}(n,\xi) = 2\Delta \beta_m(\xi) \sqrt{\frac{\mathcal{E}_{E1}^{rec}}{\alpha_m^{E1}}} \left(n + \frac{1}{2}\right), \qquad c_2^{E1}(\xi) = -\Delta \beta_m(\xi).$$
(15)

The values of all frequency-dependent quantities here are taken at the magic frequency of equal E1 polarizabilities (denoted by indexes "m" and "E1").

As is seen from equations (14) and (15), the possibility appears to introduce an "operational magic" frequency [7], which could minimize the lattice-induced clock-frequency shifts (13) within the operational distribution of the laser intensity over the lattice sites occupied by trapped atoms. To this end, the magic frequency shift  $\Delta \omega_m$  should be controllable value, together with the circular polarization degree  $\xi$  and the intensity of the lattice laser I. In this case, the contribution of  $\Delta \alpha_m^{qm}$  to the coefficient  $c_{1/2}^{E1}(n)$  and the hyperpolarizability contribution to  $c_1^{E1}(n,\xi)$  may be compensated by the E1-polarizability derivative term. Evidently, for the most efficient compensation, the atoms should be cooled down to the lowest possible oscillator quantum number n = 0. In addition, the polarization dependence of the hyperpolarizability (2) may be used. In the case of opposite signs of the components  $\Delta \beta_m^l$  and  $\Delta \beta_m^c$  the hyperpolarizability-dependent terms may be removed at the "magic ellipticity" [6] of the lattice wave  $\xi_m = \pm 1/\sqrt{1 - \Delta \beta_m^c / \Delta \beta_m^l}$ , for which the hyperpolarizability will vanish  $\Delta \beta_m(\xi_m) = 0$ . As is seen from equations (15), the coefficients  $c_{3/2}^{E1}(n, \xi_m)$  and  $c_2^{E1}(\xi_m)$  also vanish at the magic ellipticity.

For evaluation of the shift (13), the values of the frequency-dependent E1,E2-M1 polarizabilities and hyperpolarizabilities at the magic frequency should be known. We have used the Fues' model potential method for describing wavefunctions in the single-electron approximation. The details of calculations in the FMP approach and numerical data for susceptibilities of Ca, Sr, Yb, Zn, Cd and Hg atoms were presented in [3]. In this paper we report the results of calculations of the data for susceptibilities of Mg atoms. First of all, the frequency-dependent polarizabilities of the clock states were calculated in the region of the magic wavelength. The results are presented on figure 1 for the wavelength dependence of excited- (full curve) and ground-state (dashed curve) polarizabilities. The position of the curves intersection at 471.6 nm differs from the result of experimental measurements of the magic wavelength [1] by less than 1%., thus confirming satisfactory precision of calculations in the FMP approach.



**Figure 1.** Numerical data of the FMP calculations for the wavelength dependence of dynamic polarizabilities of excited (bold curve) and ground-state (dashed) Mg atom. Intersection of the curves determines the magic wavelength at  $\lambda_m \approx 471$  nm, close to the result of high-precision experimental measurement  $\lambda_m^{\text{exp}} \approx 468.46$  nm [1].

The magic frequency  $\omega_m / 2\pi = c / \lambda_m = 639.95$  THz locates in the violet region of the light spectrum, rather close to the clock frequency (see table 1). The energy of two photons  $2\omega_m$  exceeds the ionization potential of Mg atom. Therefore the hyperpolarizability is a complex value with the imaginary part determining the two-photon ionization rate, similar to the case of the group IIb atoms [3] and to the case of the blue-detuned magic wavelength of Sr atoms [9]. In table 1, the numerical data for the magic wavelength, dynamic polarizabilities and hyperpolarizabilities at the magic frequencies, the recoil energies and the blackbody radiation-induced clock-frequency shifts are presented for Mg atoms together with corresponding data for the group IIb atoms. The real parts of the hyperpolarizability components  $\Delta \beta_m^l$  and  $\Delta \beta_m^c$  of Zn, Cd and Hg atoms have opposite signs, providing the existence of the magic circular polarization degree  $\xi_m$  for which  $\text{Re} \{\Delta \beta_m(\xi_m)\} = 0$ .

As is seen from equations (14)-(15), there are four basic parameters which determine the latticeinduced shift of the clock frequency: (i) the laser intensity I, which should ensure the trapping condition  $D > 100 \mathcal{E}^{rec}$ , (ii) the vibration quantum number n, which should be reduced to its minimal value n = 0 by means of the side-band cooling, (iii) the degree of circular polarization  $\xi$ , which can eliminate or reduce to minimum the hyperpolarizability effects, (iv) the magic frequency shift  $\Delta \omega_m$ , which may reduce to minimal values the coefficients  $c_{1/2}^{E1}(n)$  of the square-root and  $c_1^{E1}(n,\xi)$  of linear in I terms. The data of table 1 allows to determine the most suitable values of  $\xi$  and  $\Delta \omega_m$  to ensure the best trapping conditions simultaneously with the lowest lattice-induced clock-frequency uncertainties, on the other hand. In figure 2 the corresponding numerical data for the shifts (14) are presented for Mg (a) and Hg (b) atoms in the regions of intensities, sufficient to ensure trapping conditions.



**Figure 2.** Laser-intensity dependence of the lattice-induced clock-frequency shifts in the region of the most suitable intensities and polarizations (linear for Mg and magic circular polarization degree  $\xi_m = \pm 0.86$  for Hg): (a) in Mg atoms at the magic frequency shifts  $\Delta \omega_m = -74.9$  MHz (curve 1), -75.0 MHz (curve 2) and -75.1 MHz (curve 3); (b) in Hg atoms at  $\Delta \omega_m = -2.0$  MHz (lower solid curve), -2.01 MHz (medium dotted) and -2.02 MHz (upper dash-dotted curve).

**Table 1.** Characteristics of atoms in optical lattices of magic wavelengths. Bold-faced are experimentally determined data on the magic wavelengths for Mg [1] and Hg [10]. The data for the MWL of Zn and Cd are evaluated theoretically. The coefficient  $v_0^{BBR}$  of the blackbody radiation-induced clock-frequency shift dependence  $\Delta v_{cl}^{BBR}(T) = v_0^{BBR}(T/300)^4$  on the temperature T of environment is given in the last line of the table.

Atom	Mg	Zn	Cd	Hg
$\lambda_m$ (nm)	468.46	406.5	414.4	362.57
$v_{clock}$ (THz)	655	969	903	1129
$\alpha_m^{E1}\left(\frac{\mathrm{kHz}}{\mathrm{kW/cm}^2}\right)$	17.5	8.11	9.76	5.70
$\Delta \alpha_m^{qm} \left( \frac{\mathrm{mHz}}{\mathrm{kW/cm}^2} \right)$	5.48	15.3	5.86	8.25
$\Delta \beta_m^{l} \left[ \frac{\mu Hz}{\left( kW/cm^2 \right)^2} \right]$	111+5.88 <i>i</i>	- 4.3+1.64 <i>i</i>	- 5.47+2.02 <i>i</i>	- 2.67+0.82 <i>i</i>
$\Delta \beta_m^{c} \left[ \frac{\mu Hz}{\left( kW/cm^2 \right)^2} \right]$	1735+ 8.69 <i>i</i>	42.6+2.45 <i>i</i>	19.5+3.01 <i>i</i>	0.94+1.21 <i>i</i>
$\frac{\Omega_m}{\sqrt{I}} \left[ \frac{\mathrm{kHz}}{\sqrt{\mathrm{kW/cm^2}}} \right]$	51.5	24.1	19.9	13.1
$\frac{\partial \left(\Delta \alpha_m^{E1}\right)}{\partial \omega} \left(\frac{10^{-9}}{\text{kW/cm}^2}\right)$	0.420	0.187	0.200	0.134
$\mathcal{E}^{rec}\left(\mathrm{kHz} ight)$	37.9	17.9	10.14	7.57
$v_0^{BBR}(\mathrm{Hz})$	- 0.424	- 0.23	- 0.22	- 0.188

Comparison between the lattice-wave induced shifts presented in figure 2 for Mg (plot a) and Hg (plot b) demonstrates significant advantage of the heavy atom: despite smaller magic frequency  $(\omega_m^{Mg} / 2\pi \approx 640 \text{ THz} \text{ against } \omega_m^{Hg} / 2\pi \approx 827 \text{ THz})$  the recoil energy for Mg exceeds almost 5 times that for Hg atoms, therefore the trapping laser intensity required for Mg is almost twice that for Hg, the latter should locate in the region of intensities near 90 kW/cm<sup>2</sup>. The uncertainty of the lattice-

induced shift due to the uncertainty of the intensity distribution over occupied by Mg atoms lattice sites between 140 and 160 kW/cm<sup>2</sup> achieves 10 mHz (between  $\Delta v_{cl}^{latt}$  =2157 and 2167 mHz) at the magic frequency shift  $\Delta \omega_m = -75$  MHz (see figure 2a). For Hg atoms this kind of uncertainty does not exceed 0.5 mHz (around  $\Delta v_{cl}^{latt} = -22$  mHz) for the intensities between 60 and 110 kW/cm<sup>2</sup> of the lattice wave of the magic circular polarization at the magic frequency shift  $\Delta \omega_m = -2.01$  MHz (see figure 2b). To reduce the  $\Delta v_{cl}^{latt}$  uncertainty to the level of 0.5 mHz, the fractional uncertainty of the operational intensity distribution over lattice sites, occupied by Mg atoms, should not exceed 2% around I = 152 kW/cm<sup>2</sup>.

## Acknowledgments

This work was supported in part by the Ministry of Education and Science of Russia in the framework of the state order on research activities in 2014-2016 (Project No 1226) and by the Russian Foundation for Basic Research (RFBR grant No 14-02-00516-a).

## References

- [1] Kulosa A P et al. 2015 Phys. Rev. Lett. 115 240801
- [2] Manakov N L, Ovsiannikov V D and Rapoport L P 1986 Phys. Rep. 141 319
- [3] Ovsiannikov V D, Marmo S I, Pal'chikov V G and Katori H 2016 Phys. Rev. A 93 043420
- [4] Taichenachev A V Yudin V I, Ovsiannikov V D, Pal'chikov V G and Oates C W 2008 Phys. Rev. Lett. 101 193601
- [5] Katori H, Hashiguchi K, Il'inova E Yu and Ovsiannikov V D 2009 Phys. Rev. Lett. 103 153004
- [6] Taichenachev A V, Yudin V I, Ovsiannikov V D and Pal'chikov V G 2006 *Phys. Rev. Lett.* **97** 173601
- [7] Katori H, Ovsiannikov V D, Marmo S I and Palchikov V G 2015 Phys. Rev. A 91 052503
- [8] Ovsiannikov V D, Palchikov V G, Taichenachev A V, Yudin V I and Katori H 2013 *Phys. Rev.* A 88 013405
- [9] Takamoto M, Katori H, Marmo S I, Ovsiannikov V D and Pal'chikov V G 2009 *Phys. Rev. Lett.* 102 063002
- [10] Takamoto M. et al. 2015 Comptes Rendus Physique 16 489