# Deep laser cooling of magnesium atoms using a $3^3P_2 \rightarrow 3^3D_3$ dipole transition



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# Deep laser cooling of magnesium atoms using a $3^3P_2 \rightarrow 3^3D_3$ dipole transition

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**Abstract**

We present the theoretical analysis of sub-Doppler laser cooling of $^{24}$Mg atoms using dipole transition $3^3P_2 \rightarrow 3^3D_3$ under two counterpropagating light waves with opposite circular polarizations (one-dimensional $\sigma^+\sigma^-$ configuration). For numerical calculations the standard semi-classical approach based on the Fokker–Planck equation for linear momentum distribution of atoms is exploited. The distributions are gained beyond the limits of slow atoms approximation and for an arbitrary light field intensity. The absence of these limits allows us to determine the optimal parameters of the light field to maximize a fraction of ultracold atoms ($T \sim 10\,\mu$K) in a whole atomic cloud. In particular, under certain conditions the fraction can reach a value of 50%. Solution of the existing problems in deep laser cooling of magnesium atoms has obvious prospects for atomic optics and quantum metrology: for instance, in designing new-generation optical frequency and time standards based on cold atoms in optical lattices.

Keywords: laser cooling, triplet states, optical frequency standard, laser spectroscopy

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Laser cooling and trapping of atoms is located at the intersection of modern laser physics and atom optics. It has been undergoing galloping progress during the past 30 years. Laser cooling has found numerous relevant applications, such as Bose–Einstein condensation, atomic nanolithography, quantum informatics and atom interferometry. Especially, it should be noted that laser cooling has great prospects for metrology. In particular, laser cooling and trapping techniques, together with modern spectroscopic methods, promise unprecedented accuracy of frequency and time standards [1].

Since 1967 the second, the base unit of time in several systems of units, has been defined based on the microwave transition in atomic caesium. Modern caesium microwave fountain-type standards have relative uncertainty of the order of $10^{-16}$ [2]. These types of standards, most likely, have reached their limit of potential accuracy. Major hopes for significantly enhancing metrological properties of frequency and time standards are connected with the optical spectral band [3] and usage of the femtosecond frequency comb [4]. In this direction, in the past ten years, increased attention has been paid to optical clocks based on laser-cooled single ions confined in electromagnetic traps [5] or on large numbers of atoms trapped in optical lattices [6]. The relative frequency uncertainty of these standards of the order of $10^{-18}$ and even below is expected [7].

For a few reasons, the main candidates for producing the new-generation frequency standards are alkaline earth and alkaline-earth-like atoms: Yb, Ca, Sr, Hg and Mg. To date, atoms of the first four elements can be effectively cooled down to the recoil energy limit [8], and even below, to obtain the Bose–Einstein condensate [9]. But for a long time researchers have not been able to reach the same success with Mg atoms [10]. At the same time, magnesium atoms have some advantages. For instance, the black body radiation (BBR) shift of the clock transition in magnesium (see figure 1) is smaller with respect to the other candidates. The strong dipole transition $3^1S_0 \rightarrow 3^1P_1$ with very short lifetime of exited state (2 ns)

 

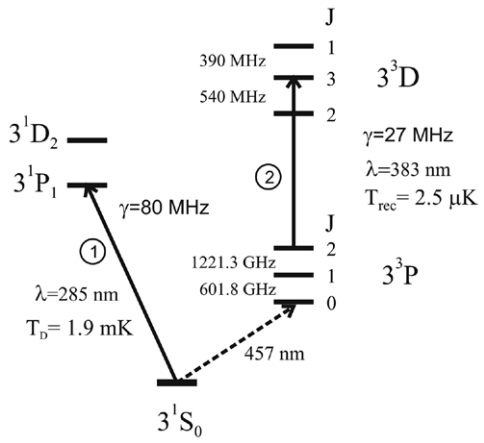


**Figure 1.** Principal energy levels of $^{24}$Mg atom. The solid lines denote the cooling transitions (the first and the second stages with corresponding temperature limits), dashed line denote highly-forbidden transition that can be used for laser stabilizing ('clock transition').

and absence of optical pumping on the non-resonant level $3^1D_2$ (in contrast to Ca and Sr) allows one to realize the first cooling stage very effectively [11–13]. Also, the relatively simple electronic configuration of the atom allows more precise calculations (for example, collisional and BBR shifts).

Recently some positive experimental results in magnesium cooling have been achieved at Hanover University [14]. The authors used $3^3P_2 \rightarrow 3^3D_3$ dipole transition for cooling and a dipole trap for collecting the ultracold magnesium atoms in the vicinity of a magneto-optical trap (MOT). However, only a small number of ultracold magnesium atoms ($N=5000$, $T=5\,\mu K$) was confined in a dipole trap. In our paper we carry out theoretical analysis of sub-Doppler laser cooling of magnesium atoms using a $3^3P_2 \rightarrow 3^3D_3$ dipole transition. The main goal of the work consists in finding the conditions which would greatly increase the number of ultracold magnesium atoms. The solution of this problem would enhance interest in magnesium atoms and provide significant prospects of cold magnesium atoms for quantum metrology and other applications.

For theoretical analysis the standard semi-classical approach is used, based on the Fokker–Planck equation for steady-state distribution of atoms in the momentum space $f(p)$. We assume the light field configuration to be one-dimensional and consisting of two counterpropagating laser beams with opposite circular polarizations ($\sigma^+\sigma^-$ configuration). To solve the problem stated we have to do calculations out of widely used approximation of slow atoms [15–19]. Being outside this limit allows us to take into account non-linear effects in light force and diffusion coefficients in their dependences on atomic velocity. It also permits investigation of the kinetic properties of a cold atomic ensemble in a wide range of light field intensities and frequency detuning.

## 2. The problem statement

Let us briefly describe the semi-classical formalism for calculating the kinetic properties of a cold atomic cloud. First of all we derive the master equation for the density of the atomic momentum distribution. Then this distribution will be used for calculating the average kinetic energy and for determining the optimal parameters to maximize the fraction of ultracold atoms in the whole atomic cloud.

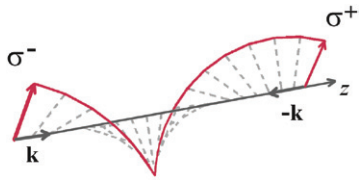


**Figure 2.** One-dimensional configuration of light field composed of two counterpropagating $\sigma^+$ and $\sigma^-$ laser beams.

As we mentioned above, the one-dimensional $\sigma^+\sigma^-$ configuration of the laser field is considered. It can be written as follows:

$$\begin{aligned}\mathbf{E}(z,\,t) &= E_0\mathbf{e}_{-1}e^{\mathrm{i}(kz-\omega t)} - E_0\mathbf{e}_{+1}e^{-\mathrm{i}(kz+\omega t)} + \kappa.c\,. \\ &= E_0\mathbf{e}(z)e^{-\mathrm{i}\omega t} + c.c., \end{aligned} \tag{1}$$

where $\mathbf{e}_{\pm1}$ are the cyclic basis vectors, connected with the Cartesian ones:

$$\mathbf{e}_{\pm1} = \mp(\mathbf{e}_x \pm \mathrm{i}\mathbf{e}_y)/\sqrt{2}\,. \tag{2}$$

The complex unit vector of local field polarization in the cyclic basis is

$$\mathbf{e}(z) = \mathbf{e}_{-1}e^{\mathrm{i}kz} - \mathbf{e}_{+1}e^{-\mathrm{i}kz}. \tag{3}$$

As follows from (1) and (2), the light field is linearly polarized and the angle between the polarization and the $x$-axis depends on the coordinate $z$: $\varphi(z) = kz$ (see figure 2).

According to the semi-classical method we should assume the characteristic width $\Delta p$ of atomic momentum distribution $f(p)$ to be much larger than the recoil momentum $\hbar k$, which an atom acquires after emitting or absorbing one photon, i.e. $\hbar k/\Delta p \ll 1$. We consider the kinetic stage of atomic cloud evolution, when the internal state of the atom can be treated as steady state. Relaxation of this state occurs during the time $t \gg \max\{(\gamma)^{-1},(\gamma S)^{-1}\}$ with $\gamma$ the spontaneous damping rate of an excited state and $S$ the saturation parameter. Also, the condition $\omega_r \ll \min\{\gamma,\gamma S\}$ should be satisfied, where $\omega_r$ is the recoil frequency. Stated assumptions are the sense of semi-classical approximation. The start equation for steady-state Wigner's distribution function has the following form (Fokker–Planck-type equation, [18, 19]):

$$\frac{p}{M}\frac{\partial}{\partial z}f(z,\,p) = \left[-\frac{\partial}{\partial p}F(z,\,p) + \frac{\partial^2}{\partial p^2}D(z,\,p)\right]f(z,\,p). \tag{4}$$

Here $M$ is an atomic mass and $F(z,p)$ is the light force acting on an atom with momentum $p$ at the coordinate $z$. This force is the quantum-mechanical average of the corresponding force operator:

$$F = \mathrm{Tr}\{\widehat{F}(z)\,\widehat{\rho}^{(0)}(z,\,p)\}\,, \tag{5}$$

where

$$\widehat{F}(z) = -\frac{\partial}{\partial z}\widehat{V}(z). \tag{6}$$

Superscript '0' to the density matrix $\widehat{\rho}^{(0)}$ means the zero order of the expansion of the matrix in the small parameter $\hbar k/\Delta p$. The operator $\widehat{V} = -\widehat{\mathbf{d}}\,\mathbf{E}$ in (6) characterizes the dipole interaction of the atom with the laser field (1). In the rotating-wave approximation (RWA) the operator $\widehat{V}$ does not depend on time:

$$\widehat{V} = -\hbar R \widehat{V}^{\,\mathrm{eg}}(z) + \mathrm{h.c.}, \tag{7}$$

with $R = dE_0/\hbar$ the Rabi frequency ($d$ is the reduced matrix element of the dipole momentum operator of an atom), h.c. means Hermitian conjugation, and the dimensionless operator $\widehat{V}^{\,\mathrm{eg}}$ determined through the Clebsch–Gordan coefficients:

$$\widehat{V}^{\,\mathrm{eg}} = \sum_{q,m_g,m_e} e_q(z)\, C^{F_e,m_e}_{F_g,m_g;1,q}\, |F_e,\, m_e\rangle\, \langle F_g,\, m_g|; \tag{8}$$

$$q = \pm 1;\ m_a = -F_a,\ \ldots,\ F_a.$$

Here $F_g$, $F_e$ are the total angular momenta of ground and excited states respectively, and the components $e_q(z)$, according to (3), are $e_{\pm 1}(z) = \mp e^{\mp ikz}$.

The function $D(z, p)$ in (4) is responsible for atomic diffusion under the light field and it can be divided into spontaneous (s) and induced (i) parts:

$$D(z, p) = D^{(\mathrm{s})}(z, p) + D^{(\mathrm{i})}(z, p). \tag{9}$$

The first term is connected with fluctuations of spontaneous photon directions, while the second term is responsible for fluctuations of the light force (for example, see [21]).

$$D^{(\mathrm{s})} = \frac{\gamma(\hbar k)^2}{5}\mathrm{Tr}\left\{\left(\widehat{P}^{\,\mathrm{e}} - \frac{1}{2}\widehat{T}_0\widehat{T}_0^{\,\dagger}\right)\widehat{\rho}^{(0)}(z, p)\right\}, \tag{10}$$

$$D^{(\mathrm{i})} = -\mathrm{Tr}\{\widehat{F}(z)\,\widehat{\rho}^{(1)}(z, p)\}. \tag{11}$$

Superscript '1' means the first order of expansion of the density matrix in the small parameter $\hbar k/\Delta p$. $\widehat{P}^{\,\mathrm{e}} = |F_e,\, m_e\rangle\,\langle F_e,\, m_e|$ is the projection operator onto the excited state of the atom and $\widehat{T}_0$ is the $z$-component of the Wigner's vector operator:

$$\widehat{\mathbf{T}} = \sum_{q,m_g,m_e} \mathbf{e}_q\, C^{F_e,m_e}_{F_g,m_g;1,q}\, |F_e,\, m_e\rangle\langle F_g,\, m_g|, \quad q = 0, \pm 1. \tag{12}$$

Steady-state equation for the atomic density matrix $\widehat{\rho}^{(0)}$, which appears in (5) and (10), is [21,22]

$$\frac{p}{M}\frac{\partial}{\partial z}\widehat{\rho}^{(0)} + \frac{\mathrm{i}}{\hbar}[(\widehat{H}_0 + \widehat{V}),\, \widehat{\rho}^{(0)}] + \widehat{\Gamma}\{\widehat{\rho}^{(0)}\} = 0, \tag{13}$$

where $\widehat{H}_0$ is the Hamiltonian of the free atom, describing internal degrees of freedom of an atom. In the RWA it can be written as

$$\widehat{H}_0 = -\hbar\delta\,\widehat{P}^{\,\mathrm{e}}. \tag{14}$$

Here we input the frequency detuning $\delta = \omega - \omega_0$ between the laser frequency ($\omega$) and the transition one ($\omega_0$). Operator $\widehat{\Gamma}\{\widehat{\rho}^{(0)}\}$ describes processes due to the spontaneous relaxation of the excited state:

$$\widehat{\Gamma}\{\widehat{\rho}^{(0)}\} = \frac{\gamma}{2}\{\widehat{P}^{\,\mathrm{e}}, \widehat{\rho}^{(0)}\} - \gamma \sum_{q=0,\pm 1} \widehat{T}_q^{\,\dagger}\widehat{\rho}^{(0)}\widehat{T}_q. \tag{15}$$

The set of the differential equations (13) on the elements of the density matrix $\widehat{\rho}^{(0)}$ is linear and homogeneous. It must be supplemented by the normalization condition:

$$\mathrm{Tr}\{\widehat{\rho}^{(0)}\} = 1. \tag{16}$$

The set of differential equations on the elements of the density matrix $\widehat{\rho}^{(1)}$ has a similar form to (13), but with a special right part (e.g., see [21]):

$$\frac{p}{M}\frac{\partial}{\partial z}\widehat{\rho}^{(1)} + \frac{\mathrm{i}}{\hbar}[(\widehat{H}_0 + \widehat{V}),\, \widehat{\rho}^{(1)}] + \widehat{\Gamma}\{\widehat{\rho}^{(1)}\} = -\frac{1}{2}\{\delta\widehat{F}(z),\, \widehat{\rho}^{(0)}(z, p)\}. \tag{17}$$

Here we input the operator of light force fluctuations $\delta\widehat{F} = \widehat{F} - F$ (see the definitions (5) and (6)).

To calculate the temperature of an atomic cloud and the rate of cooling scientists often consider only averaged functions $F(p) = \langle F(z, p)\rangle_z$ and $D(p) = \langle D(z, p)\rangle_z$, i.e. without taking into account any localization effects. In this case the $z$-coordinate can be omitted in (4). Moreover, the slow atoms approximation is also widely used [12,15–19]. According to that approximation, there is the assumption that atoms displace to a small distance (in comparison with a light wavelength $\lambda$) during the characteristic time of atomic relaxation among the internal degree of freedom. In other words:

$$\frac{v}{\lambda} << \min\{\gamma,\ \gamma S\}, \tag{18}$$

where $v$ is the atomic velocity, and the parameter of saturation $S$ is defined as

$$S = \frac{R^2}{(\gamma/2)^2 + \delta^2}. \tag{19}$$

If the condition (18) is satisfied, it is enough for analysis to be limited to linear dependence of the light force on an atomic velocity, while the diffusion may be considered independent of it:

$$F(v) \approx -\alpha v, \tag{20}$$

$$D(v) \approx D_0. \tag{21}$$

The force (20) has a friction-like form, therefore the parameter $\alpha$ is called the friction coefficient. Taking account of the above, one can easily derive from (4) the new equation:

$$\left[\frac{\alpha}{M}p + D_0\frac{\mathrm{d}}{\mathrm{d}p}\right]f(p) = 0. \tag{22}$$

Its solution is the regular dome-shaped Maxwellian distribution:

$$f(p) \propto \exp\left[-\frac{p^2}{2M\,k_B T}\right] \tag{23}$$

with the well-known expression for temperature:

$$k_B T = D_0/\alpha. \tag{24}$$

In our paper we exploit a more general approach, being out of the limitations (18) and considering atomic velocity to

be arbitrary. This allows us to study the kinetic properties of the atomic cloud for an arbitrary parameter $S$ (i.e. for any frequency detuning and light field intensity). At that, as we will see below, the momentum distribution may differ significantly from the Maxwellian one.

The equations for density matrix (13) and (17), and simultaneously the Fokker–Planck equation (4), can be significantly simplified in the case of $\sigma^+\sigma^-$ field configuration. Indeed, as follows from figure 2, the light field polarization will be constant and linearly polarized along the $x$-axis at every point of space $z$, if we choose the new coordinate system that rotates at the angle $\varphi(z) = kz$ round the $z$-axis (e.g., see [15, 23]). Thus, the interaction operator $\widehat{V}$, as well as the density matrices $\widehat{\rho}^{(0)}$ and $\widehat{\rho}^{(1)}$, the light force (5), the diffusion function (9) and the Fokker–Planck equation (4), no longer depend on the $z$-coordinate in the new coordinate system. To acquire the formulas for the new (rotating) coordinate system it is convenient to use the Wigner matrix of rotation [24], which is the representation of the rotation operator for the angle –φ around the $z$-axis:

$$\widehat{R}_z(-\varphi)\,|F,\, m\rangle = \sum_{m'} D_{m'm}(-\varphi)\,|F,\, m'\rangle\ . \tag{25}$$

In our case the rotation matrix has a simple diagonal form:

$$\langle F,\, m|\widehat{D}\,|F',\, m'\rangle = \mathrm{e}^{\mathrm{i}mkz}\delta_{mm'}\delta_{FF'}. \tag{26}$$

After transforming the matrices from the old basis to the new one, using the formula:

$$\widehat{\rho}_{\mathrm{new}} = \widehat{D}\,\widehat{\rho}_{\mathrm{old}}\widehat{D}^{*}, \tag{27}$$

we see that the density matrix elements in the new basis differ from the corresponding elements in the old basis only by a local phase:

$$\left(\widehat{\rho}_{\mathrm{new}}\right)^{F_\mathrm{a}F_\mathrm{b}}_{m_\mathrm{a}m_\mathrm{b}} = \left(\widehat{\rho}_{\mathrm{old}}\right)^{F_\mathrm{a}F_\mathrm{b}}_{m_\mathrm{a}m_\mathrm{b}}\,\mathrm{e}^{\mathrm{i}(m_\mathrm{a}-m_\mathrm{b})kz}. \tag{28}$$

In the new basis the differential equations (13) and (17) become algebraic (subscript 'new' is omitted for short):

$$\mathrm{i}\left[(\hbar k\upsilon\,\widehat{F}_z + \widehat{H}_0 + \widehat{V}_0),\, \widehat{\rho}^{(0)}\right] + \hbar\widehat{\Gamma}\{\,\widehat{\rho}^{(0)}\} = 0, \tag{29}$$

$$\mathrm{i}\left[(\hbar k\upsilon\,\widehat{F}_z + \widehat{H}_0 + \widehat{V}_0),\, \widehat{\rho}^{(1)}\right] + \hbar\widehat{\Gamma}\{\,\widehat{\rho}^{(1)}\} = -\frac{\hbar}{2}\{\,\delta\widehat{F}_0,\, \widehat{\rho}^{(0)}\}\,. \tag{30}$$

Here $\widehat{F}_z$ is the operator of the total angular momentum projection onto the $z$-axis and the operator of the atom–field interaction no longer depends on $z$:

$$\widehat{V}_0 = -\hbar R(\widehat{T}_{-1} - \widehat{T}_{+1}) + \mathrm{h.c.} \tag{31}$$

The light force operator in the new basis is also homogeneous:

$$\widehat{F}_0 = -\mathrm{i}k\hbar R(\widehat{T}_{-1} + \widehat{T}_{+1}) + \mathrm{h.c.} \tag{32}$$

Obviously, the light force (5) in the new basis does not depend on the coordinate. Taking into account the invariance of the trace operator with respect to the rotations of the coordinate system, we can state that the light force is also homogeneous in the old (non-rotated) coordinate system. This is one of the differences between the $\sigma^+\sigma^-$ configuration and the configuration lin⊥lin (orthogonal linear polarizations), where the sub-Doppler cooling mechanism also exists (so-called 'Sisyphus cooling' of atoms).

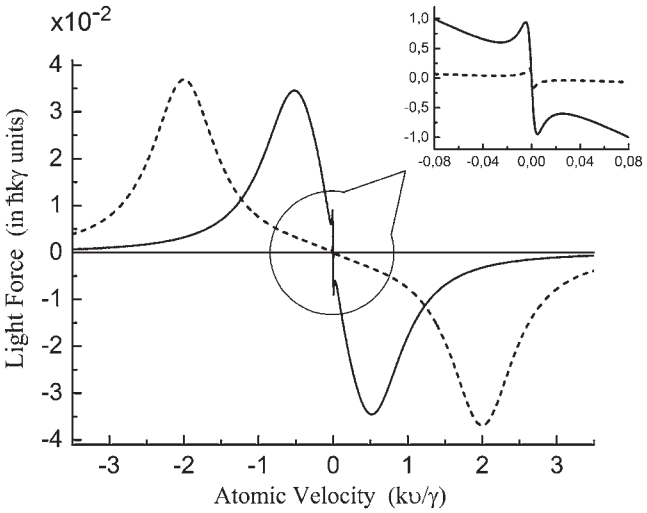


**Figure 3.** Light force under the weak laser field with $R = 0.1\gamma$. Parameters: $\delta = -0.5\gamma$ (solid) and $\delta = -2\gamma$ (dashed).

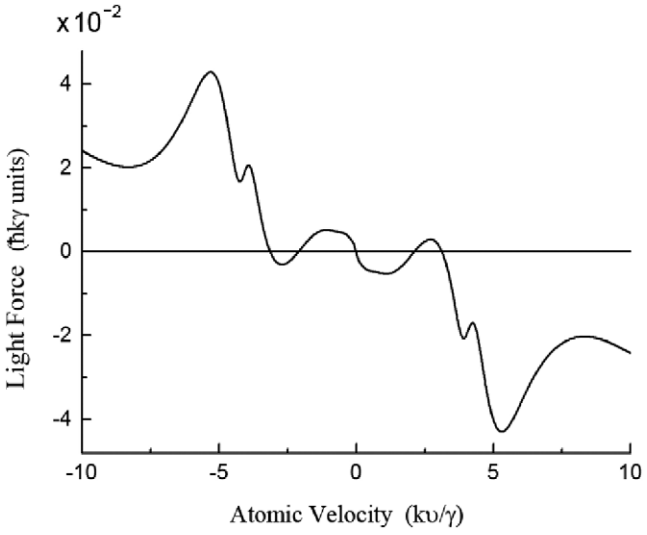


**Figure 4.** Light force in strong field regime, $R = 10\gamma$, $\delta = -0.5\gamma$.

## 3. Results and discussions

### *3.1. Light force and diffusion*

Briefly, in this paper we omit the results of a detailed study of light force and the diffusion of atoms in a laser field. We just show a few plots that demonstrate the non-linear dependence of the kinetic coefficients (force and diffusion) on atomic velocity in various regimes. Those plots have been obtained by the numerical solutions of equations (29) and (30).

Figures 3 and 4 present the dependence of the light force on an atomic velocity (Doppler shift $k\upsilon$) for weak and strong laser

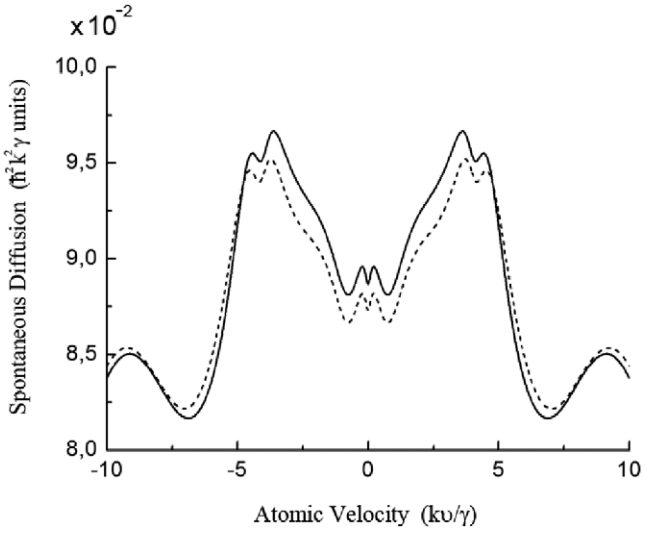


**Figure 5.** Spontaneous diffusion of atoms under strong laser field, $R = 10\gamma$, $\delta = -0.5\gamma$ (solid) and $\delta = -2\gamma$ (dashed).

fields, driving the dipole transition $3^3P_2 \rightarrow 3^3D_3$ (see figure 1). As seen in figure 3, the light force has typical resonance peaks in the vicinity of the values $k\upsilon = \pm\delta$. Far from the zero group of atomic velocity ($k\upsilon > \gamma$) the counterpropagating $\sigma^+$ and $\sigma^-$ fields act on the atoms almost independently. Therefore, in the resonant velocity groups $k\upsilon = \pm\delta > \gamma$ in the steady-state atoms are accumulated on one of the last Zeeman sublevels $|F_g = 2, m_g = \pm 2\rangle$ (due to optical pumping). This situation is similar to a non-degenerate two-level atom and the estimate for the light force can be acquired via a simple equation for the force which acts on the two-level atom under a traveling lightwave of weak intensity [18, 19]:

$$F_{\max} \approx \hbar k\gamma\, S(\delta = \mathbf{k}\upsilon) = \hbar k\gamma \frac{R^2}{\gamma^2/4} = 0.04\,\hbar k\gamma. \qquad (33)$$

This value is close to that given by the exact numerical solution (see figure 3). At the centre of the plot a sharp dispersion-like structure is observed, which is the manifestation of the sub-Doppler mechanism of laser cooling under the laser field with polarization gradients [15]. The presence of this structure leads to observation of sub-Doppler temperatures of cold atoms. However, it should be stressed that momentum distribution of atoms is defined not only by that linear part of the light force function, but also non-linear parts which may also affect the distribution in a significant manner (even under weak light field intensity). Therefore, the non-linear dependences in the force and diffusion functions may result in a complex shape of the distribution (as demonstrated below). It is very important to be able to control the shape of the momentum distribution of atoms, because it gives us the opportunity for effective concentration of the cold atoms in the required regions of the momentum space (for example, in the vicinity of ultralow motions $\upsilon = 0$).

The force exhibits wavy transformations with the light intensity increasing (figure 4). These additional resonances result from non-linear atom–field interactions, when induced multiphoton absorption and emission processes become considerable (e.g., see [18, 19]). Moreover, the light force and

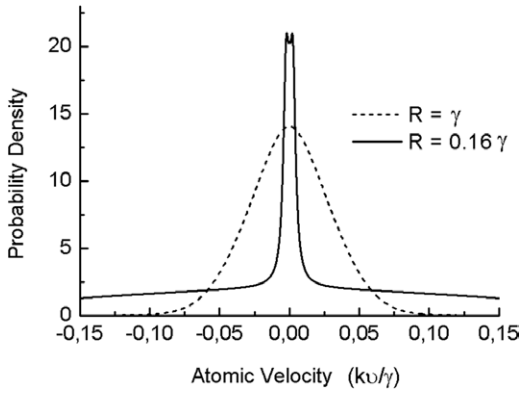


**Figure 6.** Velocity distributions of atoms for $R = 0.16\gamma$ (solid) and $R = \gamma$ (dashed), $\delta = -2\gamma$.

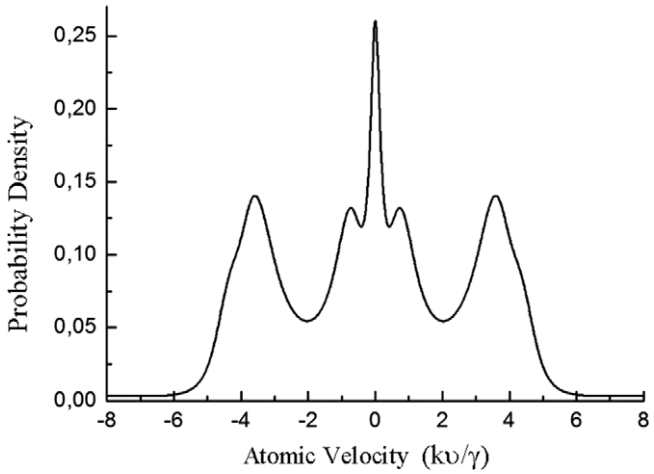


**Figure 7.** Velocity distribution of atoms for the case of strong light field $R = 10\gamma$, $\delta = -0.1\gamma$.

the $k\upsilon$-axis may intersect not only at the zero point, which leads to localization of atoms in the momentum space in the vicinity of various points $k\upsilon \neq 0$. The diffusion coefficients (9) also exhibit large non-linearity with increasing intensity. We just show the spontaneous part of total diffusion in figure 5 to demonstrate this fact. The induced part of the diffusion also suffers from strong non-linear effects.

### *3.2. Average kinetic energy and a fraction of ultracold atoms*

When the momentum distribution function $f(z, p) = f(p)$ does not depend on the $z$-coordinate the Fokker–Planck equation (4) can be written in the following manner:

$$2\omega_r D(u) f'_u(u) + [2\omega_r D'_u(u) - F(u)]\, f(u) = 0. \qquad (34)$$

with $u = k\upsilon/\gamma$. To derive this formula we have assumed physically meaningful boundary conditions $f(\pm\infty) = 0$. All quantities in (34) are dimensionless: recoil frequency $\omega_r = \hbar k^2/2M\gamma$, light force is written in $\hbar k\gamma$ units, and diffusion in $(\hbar k)^2\gamma$ units. The solution of (34) can be written in quadrature form:

$$f(u) = C \cdot \exp\left[\int_{-\infty}^{u} \frac{F(\tilde{u}) - 2\omega_r D'_{\tilde{u}}(\tilde{u})}{2\omega_r D(\tilde{u})}\, \mathrm{d}\tilde{u}\right]. \qquad (35)$$

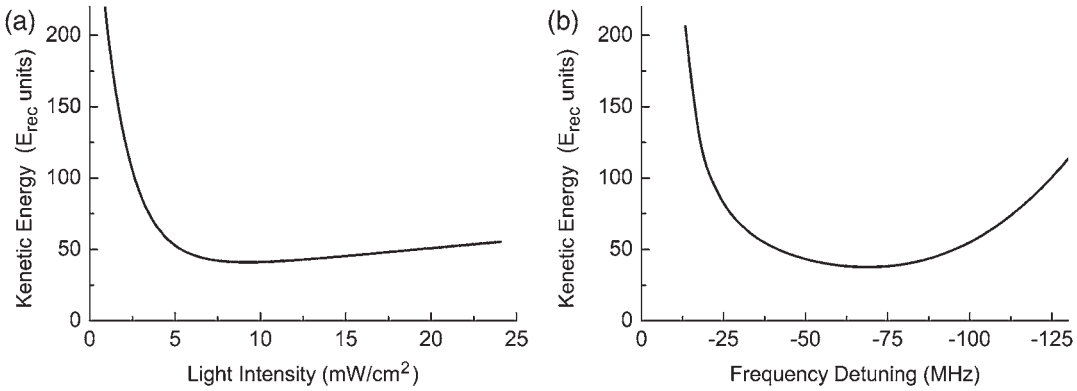


**Figure 8.** Averaged kinetic energy of atomic ensemble as a function of (*a*) light field intensity for $\delta = -2\gamma$, and (*b*) as a function of frequency detuning for $R = 0.7\gamma$.

The constant should be chosen taking into account the normalization condition:

$$\int_{-\infty}^{\infty} f(u)\,\mathrm{d}u = 1. \tag{36}$$

Further we show several plots of the velocity distribution gained by the numerical solving of equation (34). It is seen from figure 6 that velocity distribution can have a significantly non-Maxwellian form, in contrast to the situation that happens when we use the slow atoms approximation (see formula (23)). If we worked under the limits of that approximation, we would gain a narrow part of the distribution and we could expect that all the atoms are ultracold. But the calculations made out of the slow atoms approximation provide a more realistic shape for the distribution.

The wide substrate of the distribution in figure 6 results from the Doppler cooling mechanism, while the narrow one is led by the sub-Doppler part of the light force (linear narrow part at the centre of figure 3). The distribution is broadened and the atoms get hotter when the light field intensity increases. Also the intensity increasing can lead to observation of the additional points of localization of cold atoms in velocity space (figure 7). These peaks, obviously, are connected with the peculiarities of the light force that have been mentioned above (see also figure 4).

Since velocity distribution in general has a non-Maxwellian form, it is more correct to consider the average kinetic energy of the ensemble instead of the temperature (unlike the case of slow atoms approximation). Figures 8(*a*) and (*b*) present the dependences of average kinetic energy on light intensity (Rabi frequency squared $R^2$) and frequency detuning, respectively. Let us note that in the method based on the slow atoms approximation we would not acquire such dependences with extrema (more precisely the minimum values of the functions). So, only the calculations out of that limit provide an opportunity to establish optimal parameters for cooling of atoms.

By analysing the minima on the plots of figures 8(*a*) and (*b*) in their dependences on the frequency detuning and laser field intensity, we can conclude that the minimum average kinetic energy of the ensemble that can be achieved is about $30 \times E_{\mathrm{rec}}$. Roughly speaking, it corresponds to the effective temperature

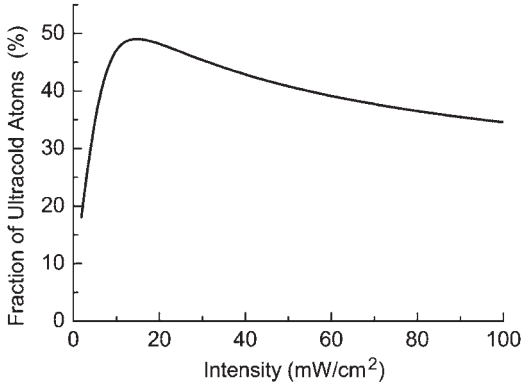


**Figure 9.** Fraction of ultracold atoms in the cold atomic cloud, $\delta = -2\gamma$.

$T_{\mathrm{eff}} = 30 \times 2.5\,\mu\mathrm{K}$. It means that our cooling method, based on the transition $3^3\mathrm{P}_2 \rightarrow 3^3\mathrm{D}_3$, does not provide a sufficiently low temperature of the atomic cloud, just a bit less than 100 μK. The experimental result of [14] can be considered as the indirect confirmation of our calculations, because the authors of that work were able to observe only a small number of ultracold atoms, while the temperature of the ensemble as a whole was not less than 100 μK. However, for many further applications of cold atoms, including metrological ones, much colder magnesium atoms are required and in significant numbers.

Thus, a new task arises: how can we maximize the fraction of ultracold atoms in the whole ensemble? This problem can be solved by analysing the velocity distributions under various conditions instead of studying the average kinetic energy (or effective temperature) of the ensemble as a whole. As is seen in figure 6, the special shape of the distribution can be made by concentrating many of the atoms in the central part of the curve. Let us see what fraction of the magnesium atom will have a momentum, for instance, of $p < 3\hbar k$. Figure 9 shows that these special conditions can be achieved by increasing the number of ultracold ($p < 3\hbar k$) atoms up to 50%, which is rather a big value (the total number of atoms in a cloud may equal $10^6$–$10^8$). After the cooling process in the MOT under these optimal conditions there will be ultracold as well as cold atoms in the whole cloud (e.g., see figure 6). We can collect only the ultracold part of the atoms by exploiting, for instance, a dipole trap with a certain depth, so that the relatively 'hot' atoms will leave the trap. Then we get an atomic cloud with many atoms ($N \sim 10^5$) at a very low

effective temperature (1–10 μK). So, this is the sense of a method for getting a large number of ultracold magnesium atoms.

In conclusion we would like to note some important details. Even though the dipole transition $3^3P_2 \rightarrow 3^3D_3$ is cyclic, nevertheless there is some probability that atoms will populate $3^3D_1$ and $3^3D_2$ levels. This is because these levels are not so distant from $3^3D_3$ (see figure 1). From the levels $3^3D_1$ and $3^3D_2$ an atom can spontaneously decay to the ground levels $3^3P_0$, $3^3P_1$ and $3^3P_2$. Only the level $3^3P_2$ is resonant with the cooling lasers. So, additional repumping laser fields must be applied to pump the atoms from the non-resonant levels to the level $3^3P_2$ for achieving effective cooling conditions for the cloud.

Also, we would like to note that we have considered 1D configuration of the light field. As the same time, the real 3D configuration, which is used for cooling in the MOT, being much more complicated for theoretical analysis, may lead to some changes in comparison with the 1D one (for example, see [12, 25]). So, the calculated values of optimal parameters may also be changed. Nevertheless, we consider that the main idea of the paper may be applicable to the 3D field as well.

The last thing we should mention is that the calculations for very cold atoms ($p \sim \hbar k$), strictly speaking, should be based on a more exact equation than the Fokker–Planck equation (4), which takes into account the recoil effect only in the first order of perturbation theory. Thus, we are going to carry out the exact quantum analysis of the problem in our next paper, based on the method developed in the literature [26].

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