Broadband laser cooling on narrow transitions

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Received April 3, 1989; accepted June 26, 1989

Laser cooling on transitions with a linewidth Γ much narrower than the recoil shift $\hbar k^2/m$ is studied numerically according to a full quantum-mechanical treatment of the photon recoil. We propose and investigate a broadband spectral configuration that has the purposes of (1) shortening the long cooling time characteristic for monochromatic laser cooling on a slow transition and (2) reducing the kinetic energy of the stationary momentum distributions considerably below the one-photon recoil energy $(\hbar k)^2/2m$. The energies of the computed stationary distributions are limited only by the energy uncertainty of the narrow transition $\hbar\Gamma \ll (\hbar k)^2/2m$. Applications to laser cooling of alkaline earths on the narrow intercombination lines are discussed.

1. INTRODUCTION

Cooling of free neutral atoms by near-resonant laser radiation has been successfully used to reduce their kinetic energy down to the energy width of broad allowed dipole transitions,[1] corresponding to temperatures in the range below 1 mK. Recent experiments have shown effects of deep cooling down to a few microkelvins that are closely connected to the response of the internal multilevel dynamics of the atoms moving in spatially varying polarization gradients.[2]

In contrast to this situation, we investigate here one-dimensional laser cooling of V-level atoms in counterpropagating σ^+–σ^- radiation, which is determined only by spontaneous scattering of photons. Induced light forces are excluded because angular-momentum conservation prevents redistribution of photons among the counterpropagating waves.[3] Preferably alkaline earths with their broad ($^1S_0 \rightarrow {}^1P_1$) and narrow ($^1S_0 \rightarrow {}^3P_1$) transitions may be cooled by this scheme.

The new feature of our investigation is to consider the case of a narrow atomic linewidth that makes a fully quantum-mechanical treatment of the photon recoil necessary. Castin *et al.* used such a treatment for determining the limits of Doppler cooling independently of the atomic linewidth[4]; we apply their ansatz to study cooling below the recoil limit down to the nanokelvin range.

Let us recall at this point the semiclassical picture of laser cooling on broad transitions and its limits[5]: in a red detuned standing wave the Lorentzian rate of absorption of photon momentum from the counterpropagating wave increases linearly with the momentum p near $p = 0$ (this gives damping), while the total scattering rate is to the first order independent of momentum (this gives constant rate of fluctuation owing to spontaneously emitted photons). For broad lines the counterbalance of force and diffusion results in a Maxwellian steady state limited by $\langle p^2/2m\rangle \approx \hbar\Gamma/4$ as a lower boundary for the final momentum spread.

In contrast to this, our treatment does not consider the average force and diffusion coefficient near $p = 0$ but rather an integral momentum-balance equation for the steady-state distribution whose width may be much smaller than the recoil width. Cooling on a narrow transition with *monochromatic* radiation is then found to be limited by the recoil energy: the spread $\langle p^2\rangle$ of the narrowest possible distribution equals $(\hbar k)^2/2$.[4] With increasing linewidth the minimum kinetic energy converges to the broad-line limit.

The quantum treatment also yielded restrictions on the detuning appropriate for cooling. If the laser is detuned into resonance with velocities smaller than $(21/20)v_R$, where $v_R = (\hbar k)/m$, no normalizable stationary solution of the momentum-balance equation exists.[4] For these small detunings, single spontaneous recoils shift the velocity out of the velocity range defined by the resonance velocities with respect to either of the counterpropagating waves; on the average atoms with kinetic energy near the recoil energy gain more momentum than they loose, and as a consequence the spread of the probability distribution is unlimited for an improper detuning. The described escape of probability manifests itself as non-Maxwellian wings of the stationary distribution for small (but allowed) values of the detuning.

To avoid this kinematic restriction on the detuning and to compress the momentum distribution much below the recoil limit we propose to use *multichromatic* excitation[6] of the Doppler range in order to repump atoms to velocities smaller than a certain cutoff velocity, which is defined by the blue cutoff frequency of the spectral profile of the incident radiation. A similar scheme has been discussed under the assumption of an idealized square excitation profile and zero linewidth by Pritchard *et al.*[7]; white-light cooling has been investigated both theoretically and experimentally for laser cooling on broad transitions.[8]

The multichromatic excitation profile is strongly supported by an additional motivation that becomes important in the case of narrow transitions: the extremely low scattering rate on a narrow transition leads to unachievable long cooling times for atoms with initial velocities larger than the recoil velocity. A confining external force (whether optical or not) preventing the spatial diffusion during the long interaction time could not be applied because it would probably destroy the low temperatures by heating owing to trapping-force fluctuations.

The problem is thus to design a molasses-type cooling experiment that may cool a velocity distribution much wider than the cooled velocity range accessible by single-line cooling. This is in principle comparable to the efforts to decel-

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erate atomic beams over a velocity region much larger than the typical Doppler velocity $v_D = \Gamma/k$.

In the case of atomic beams there do exist two well-established techniques to address the problem of maintaining resonance during deceleration that are not applicable to monochromatic laser cooling in the narrow-linewidth case. With the first technique one scans the laser frequency over the Doppler width by an adiabatic frequency chirp.[9] For our problem of cooling on a slow transition any frequency chirping at a rate $\Gamma\hbar k^2/m$ would be too fast to allow the atoms to lock to a fixed value of the Doppler detuning, thus they would not be slowed down without loss from resonance with the chirped laser.[10] In addition, the pulsed working mode of the first solution would be unfavorable since the achievable densities are small. The second technique relies on position-dependent Zeeman tuning of the transition into resonance.[11] It would include positional trapping and for the molasses situation would be equivalent to the magneto-optical trap realized in Ref. 12. For narrow resonance cooling this trapping scheme would suffer from a much larger leakage rate than in the broad-line case, therefore an application of multichromatic excitation would also be favorable.

For both purposes—stronger compression of the distribution and fast cooling of a large velocity range—we assume that applying broadband radiation pressure by a set of narrow laser sidebands tuned to the red of the resonance is a technique with unique possibilities in the case of narrow transitions. The aim of the present paper is to discuss such a cooling scheme in a quantitative way consistent with an exact quantum-mechanical description of the problem.[4] Up to now, to our knowledge no analytical methods have been applied to the study of multichromatic excitations. In order to discuss realistic experimental conditions we here employ a numerical approach.

The structure of the paper is as follows: In Section 2 we summarize the features of the quantum treatment of the photon recoil that are essential for our calculations and introduce the momentum-balance equation and discuss its significance for narrow-transition laser cooling by arbitrary spectral schemes. In Section 3 we characterize the multichromatic spectral configuration and demonstrate numerically that, by using it, a minimum kinetic energy of the order of $\hbar\Gamma \ll (\hbar k)^2/2m$ can be reached. We also discuss the multiline configuration in view of cooling alkaline-earth elements on the intercombination transition in limited interaction times.

For monochromatic cooling our results are consistent with the numerical and analytical results of Ref. 4 and seem to agree in the single-line case with Monte Carlo simulations recently performed independently by Phillips *et al.*[13]

2. QUANTUM DESCRIPTION OF LASER COOLING ON NARROW TRANSITIONS

Semiclassical descriptions of radiation pressure[5] are based on the assumption that the atomic momentum remains constant during the scattering process and enters the density-matrix equations as a parameter, whereas radiation-induced momentum changes can be found in a perturbative way in powers of the photon momentum $\hbar k$. For narrow transitions, however, the momentum range within resonance, $\Delta p \approx m\Gamma/k$, is much smaller than $\hbar k^2/m$, and the quantum-mechanical density flux in momentum space can only be treated in an integral form that is exact to arbitrary order in $\hbar k$.

Broadening of the atomic response owing to laser fluctuations is completely neglected in this paper but may be included in a future publication. The multichromatic spectral profile is always assumed to be realized by means of sidebands of an ideal monochromatic laser.

A. Determination of the Steady State

We investigate atoms with a $(J = 0 \rightarrow J = 1)$ transition interacting with counterpropagating σ^+–σ^- laser waves (see Fig. 1). In this configuration the coherent interaction with the polarized laser fields couples only closed families[4,14] of momentum states. The family consists of the ground state $|g, p\rangle$ (p is the component along the laser-beam axis) having $m = J = 0$ and the two excited states with a momentum shifted by $\pm\hbar k$ and the magnetic quantum number m by ± 1, $|e^+, p + \hbar k\rangle$ and $|e^-, p - \hbar k\rangle$. The atomic evolution due to induced processes within one family is then described by a particularly simple density-matrix equation of the general form $\mathrm{d}\rho(p)/\mathrm{d}t = L(p)\rho(p)$ because the exchange of the photon momentum with the laser field has been absorbed into the definition of the density-matrix variables (including the rotating-wave approximation):

$$\begin{aligned}
\pi_g(p) &= \langle g, p|\rho|p, g\rangle,\\
\pi_+(p) &= \langle e^+, p + \hbar k|\rho|e^+, p + \hbar k\rangle,\\
\pi_-(p) &= \langle e^-, p - \hbar k|\rho|e^-, p - \hbar k\rangle,\\
\rho_{g+}(p) &= \langle g, p|\rho|e^+, p + \hbar k\rangle \exp(i\omega_L t),\\
\rho_{-+}(p) &= \langle e^-, p - \hbar k|\rho|e^+, p + \hbar k\rangle. \qquad (1)
\end{aligned}$$

Note, e.g., that the excited-state populations having an expectation value of the momentum $p \pm \hbar k$ are labeled by p.

The interaction with the vacuum field, i.e., the spontaneous emission, now modifies this evolution in an essential way: The relaxation operator introduces the only term that is nondiagonal in the family momentum p and describes the probability transfer due to spontaneous decay into a momentum class p from momenta p' between $p + \hbar k$ and $p - \hbar k$. This contribution to the evolution of the ground-state density is

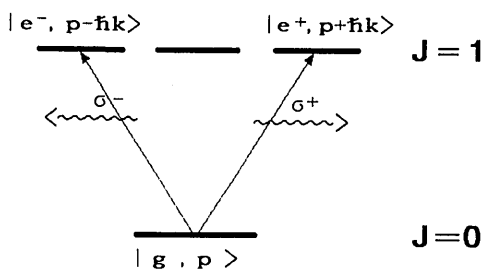


Fig. 1. Scheme of a $(J = 0 \rightarrow J = 1)$ transition driven by plane counterpropagating σ^+–σ^- laser waves. The state $|e, m = 0, p\rangle$ is not coupled to the ground state.

$$[\mathrm{d}\pi_g(p)/\mathrm{d}t]_{\mathrm{SE}} = \Gamma \int_{p-\hbar k}^{p+\hbar k} [\pi_+(p' - \hbar k) + \pi_-(p' + \hbar k)] \times \Phi(p - p')\mathrm{d}p', = : \Gamma[\overline{\pi_+(p - \hbar k)} + \overline{\pi_-(p + \hbar k)}], \quad (2a)$$

where

$$\Phi(p) = [1 + (p/\hbar k)^2]3/8\hbar k \quad (2b)$$

is the one-dimensional projection of the dipole emission pattern onto the direction of the laser standing wave after summation over the transverse momentum degrees of freedom. If we include this modification, the relevant density-matrix equations are

$$\mathrm{d}\pi_g(p)/\mathrm{d}t = \Gamma[\overline{\pi_+(p - \hbar k)} + \overline{\pi_-(p + \hbar k)}] - i\Omega[\rho_{+g}(p) - \rho_{g+}(p)]/2 - i\Omega[\rho_{-g}(p) - \rho_{g-}(p)]/2, \quad (3a)$$

$$\mathrm{d}\pi_+(p)/\mathrm{d}t = -\Gamma\pi_+(p) + i\Omega[\rho_{+g}(p) - \rho_{g+}(p)]/2, \quad (3b)$$

$$\mathrm{d}\rho_{g+}(p)/\mathrm{d}t = i\Omega[\pi_g(p) - \pi_+(p) - \rho_{-+}(p)]/2 - [i(\delta' - kp/m) + \Gamma/2]\rho_{g+}(p), \quad (3c)$$

$$\mathrm{d}\rho_{+-}(p)/\mathrm{d}t = i\Omega[\rho_{+g}(p) - \rho_{g-}(p)]/2 - (2ikp/m + \Gamma)\rho_{+-}(p), \quad (3d)$$

where Ω is the Rabi frequency $\mathrm{d}E_0/\hbar$ of each traveling wave. We remark that the correct detuning term appearing in the density-matrix equation includes the recoil energy $(\hbar k)^2/2m$ of the excited state,

$$\delta' = \omega_L - \omega_A - \hbar k^2/(2m). \quad (4)$$

δ', the dressed detuning, determines the resonant velocity according to $v = \delta'/k$.

Our model system represents the ideal scattering force experiment, where there is no induced radiation pressure because redistribution of the momentum between the counterpropagating laser wave is excluded. The kinematic transfer of probability in momentum space is caused by the incoherent contribution to the evolution of the ground state only [Eq. (3a)]. This simple connection between internal and external dynamics is assumed for the broadband excitation scheme as long as it affects only the excited populations in a different way.

To understand the time evolution in the case of a slow transition we have to keep in mind that the internal evolution does *not* reach its steady state on a time scale much shorter than the time scale of radiation-pressure-induced momentum changes, as is the case for fast transitions (broad lines). Therefore exact time-dependent solutions would involve *all* density-matrix variables. Thus to obtain the total steady state we have to solve the equation $\mathrm{d}\rho/\mathrm{d}t = 0$ for the internal and external degrees of freedom simultaneously. The integral condition for the steady state of the ground-state population is obtained from Eqs. (3a) and (3b):

$$0 = \mathrm{d}\pi_g(p)/\mathrm{d}t = \Gamma\left\{-\pi_+(p) - \pi_-(p) + \int_{p-\hbar k}^{p+\hbar k} [\pi_+(p' - \hbar k) + \pi_-(p' + \hbar k)]\Phi(p - p')\mathrm{d}p'\right\}. \quad (5)$$

Equation (5) is the basis for our numerical computation of the stationary momentum distribution in the multichromatic case as well. The integral represents the correct averaging over the excited atoms without any expansion in p.

For a direct solution of Eq. (5) it is necessary to know the excited-state populations. If only a single monochromatic sideband is present, $\pi_+(p)$ and $\pi_-(p)$ can be obtained exactly as functions of the population $\pi_g(p)$. The resulting steady-state excited-state fraction $\pi_+(p)/\pi_g(p)$ is given in Appendix A and converges for low intensity to a Lorentzian. Let us emphasize that this substitution,

$$\pi_+(p) = \gamma_+(p)\pi_g(p), \quad (6)$$

involving only the ground-state population at momentum p, is actually a particular and important feature of our model system. Usually such a relation would be correct only to zeroth order of $\hbar$ in a semiclassical iterative treatment. Equation (6) is the reason for the agreement of Monte Carlo calculations with numerical solutions of the exact density-matrix equations $\mathrm{d}\rho/\mathrm{d}t = 0$. Specific quantum states such as coherent superpositions of momentum eigenstates[14,15] are absent in this system.

Insertion of Eq. (6) into Eq. (5) yields the one-component integral equation:

$$0 = -\gamma_+(p)\pi_g(p) - \gamma_-(p)\pi_g(p) + \overline{\gamma_+(p - \hbar k)\pi_g(p - \hbar k)} + \overline{\gamma_-(p + \hbar k)\pi_g(p + \hbar k)}. \quad (7)$$

Equation (7) is the master equation for the steady state of momentum-space optical pumping[7] below the recoil momentum using narrow transitions. Neither the excitation function nor the momentum distribution has to be assumed to be smooth. For monochromatic cooling on narrow transitions the cooling limit could be derived analytically from Eq. (7).[4] However, for our purpose of investigating broadband radiation pressure on transitions with narrow linewidths we compute the stationary momentum distributions numerically and by a Monte Carlo method using Eqs. (6) and (7) plus the normalization condition. Discretization of the momentum and solution of the resulting linear system of equations yielded all curves plotted continuously. In the broadband case the necessary normalizability of the ground state was inferred from the fact that the $(v \to \infty)$ part of the distribution is suppressed by the broadband radiation pressure for any negative detuning.

Alternatively we used a little memory-consuming Monte Carlo calculation to determine the steady-state momentum distribution. In this simulation, for a given momentum p the excitation rates are calculated according to the steady-state probability $\pi_+(p)$, while the deexcitation is determined by spontaneous decay in a low-intensity approximation. The actual time spent between successive quantum jumps is then chosen as a Poisson-distributed random number according to these rates. The component of the fluorescence recoil momentum on the direction of the laser beam is chosen according to the distribution Eq. (2b). Provided that a stationary solution exists, both internal and external dynamics equilibrate for $t \to \infty$, and therefore the momentum probability distribution of a random ensemble of atoms will converge to the exact solution within the statistical error after a sufficient time, depending on the saturation and the initial distribution.

The agreement of the Monte Carlo algorithm with the direct solution of Eq. (7) was tested over a wide range of parameters. It is therefore a versatile tool for the analysis of more complex experimental situations. Since we are not interested in the study of any short-time coherent transients of the atomic dynamics, which is described by a full time-dependent solution of Eqs. (3), we make use of the Monte Carlo approach not only for the steady state but also for the study of the kinetic stage of cooling in *finite* interaction time, which is generally much longer than the lifetime of the excited state.

B. Features of the Final Momentum Distribution

We now briefly discuss the results concerning the specific limitation of monochromatic laser cooling.[4] The minimum kinetic energy achievable by monochromatic laser cooling on narrow transitions is approximately $(\hbar k)^2/4m$. In contrast with the broad-line case, for narrow lines the minimum momentum spread is not reached for $\delta' = -\Gamma/2$ but for $\delta' \approx -2.2kv_r$. The fact that the detuning $-2.2kv_R$ is optimum in the single-line case can be understood in terms of velocity-space optical pumping: The dramatic variation of the narrow absorption profile with respect to the velocity within the range $(-v_R, v_R)$ prevents any form of linear damping. Cold atoms tunnel out of the peak by a single scattering process for improper detuning. The escape process generating the non-Gaussian wings of the stationary distribution can only be avoided if $\delta' < -2\hbar k^2/m$, which is an inherent limitation of monochromatic narrow-transition laser cooling.

The shape of the final distribution is not Gaussian in general but depends on the detuning. Therefore the distribution is not sufficiently characterized by $\langle p^2\rangle$ as a single parameter. Likewise the notion of a temperature as the single relevant parameter of a Boltzmann ensemble is not adequate for the final state of the cooling process. Thus the definition of an optimum cooling result or of a quantum limit of laser cooling, respectively, requires noting the criterion chosen. This might be the height of the distribution at $p = 0$, its HWHF, or its rms momentum. Usually the distribution with maximum height is found for detunings smaller than those that produced a minimum rms velocity or the smallest HWHM velocity. The detuning that is appropriate for a minimum quadratic momentum spread is at the same time large enough to produce a more Gaussian distribution shape.

As a consequence we define the cooling limit of the broadband configuration by the 85th percentile velocity as well as by the rms velocity of the final distribution.

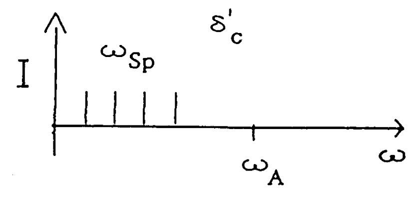
3. LIMITS OF MULTICHROMATIC LASER COOLING

The limitations of laser cooling on narrow transitions can be overcome if a well-defined broadband excitation profile is applied. It may be provided by a set of narrow laser sidebands whose intensity envelope is reduced to zero within a sharply defined frequency interval at a frequency resonant to a certain cutoff velocity [Fig. 2(a)]. Thus large-velocity admittance is combined with the high-velocity resolution made possible by the narrowness of the atomic transition.

A. Description of the Multichromatic Spectral Scheme

For a study of broadband cooling we apply Eq. (7) using a low-intensity approximation of the single-line excitation function by a Lorentzian. The agreement between the Lorentzian and the exact excitation function is sufficient for larger detunings. For narrow lines the appearance of a large detuning such as $|\delta| \approx kv_R > \Gamma$ gives a situation for which the approximation is good. The total rates of absorption from the right and the left laser beams are then estimated by incoherent addition of the excitation rates for monochromatic excitation. Intermodulation of the excitation on the scale of the difference frequency of the lines is ignored for this investigation.

We set



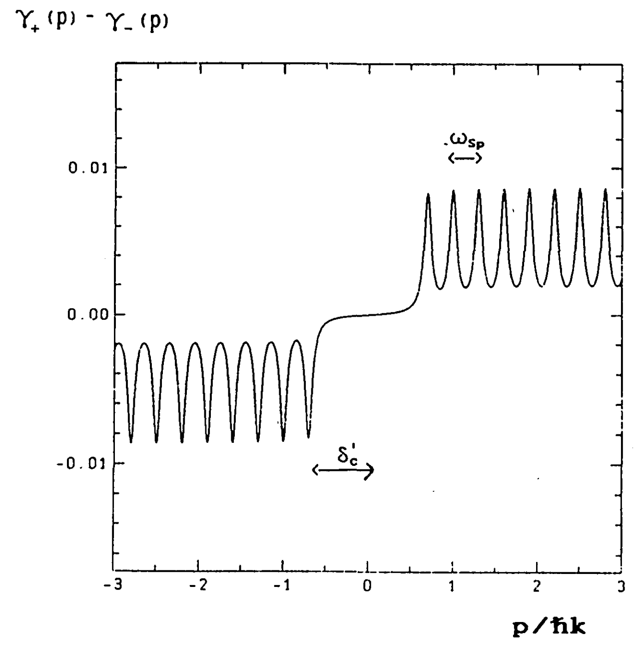


Fig. 2. Principle of the multiline excitation profile. (a) The laser spectrum; (b) the dimensionless difference $\gamma_+ - \gamma_-$ of the excited-state fractions in the low-intensity limit corresponding to the difference of the absorption probability from the two counterpropagating waves. The characteristic parameters of the profile are the cutoff frequency δ_c', the frequency spacing ω_{sp}, and the number of sidebands N. The extension of the frequency profile may be chosen according to the desired momentum admittance.

$$\gamma_+(p) = \sum_1^N \gamma_{+n}(p), \tag{8}$$

where the excitation probability γ_{+n} for the nth line tuned with a detuning $\delta_n' = \delta_c' - (n-1)\omega_{sp}$ is

$$\gamma_{+n}(p) = \frac{P_0}{1 + (\delta_n' - kp/m)^2/(\Gamma/2)^2}, \tag{9}$$

where P_0 is the saturation parameter $(\Omega/\Gamma)^2$ for a single sideband.

A plot of this incoherently summed excitation profile is shown in Fig. 2(b). Of course, the incoherent generalization has to be restricted to low saturation and should not be considered as a substitute for future exact treatments. If the multiline spectrum is realized by electro-optic phase modulation, a study of the phase-modulated Bloch equations might be the starting point. Since we focus here on the kinetic aspect of the problem we assume that a simulation in the low- and intermediate-saturation limit is justified.

B. Optimum Cooling Parameters

Reaching the quantum limit of broadband laser cooling requires a careful determination of the optimum spectral parameters, i.e., frequency spacing ω_{sp}, frequency range, and cutoff detuning δ_c'. The main purpose of the multiline scheme—to repump atoms from the non-Gaussian wings of the momentum distribution—can be qualitatively demonstrated already with a small number of sidebands. The steady-state distributions plotted in Fig. 3(a) show clearly the additional compression of the distribution and the suppression of the $(v \to \infty)$ wings owing to repumping of atoms. Figure 3(b) shows a Monte Carlo result in perfect agreement with the numerical solution of Eq. (7) plotted in Fig. 3(a) for $N = 4$. With an increasing total spectral width [Fig. 3(c), with increasing N], a reduction of the final momentum spread is achieved owing to the suppression of the wings of the distribution.

Figure 3(c) is an example of the result (which is valid for any values of spacing and detuning), showing that an overall momentum range Δp of $2\hbar k$ at minimum has to be covered in order to reach an optimum compression of the final distribution. This corresponds to the fact that atoms excited at $p = 0$ can decay to momenta $p = \pm 2\hbar k$ at maximum. Of course, in real experiments a momentum admittance larger than $2\hbar k$ is desired, e.g., to cool alkaline earths in a two-step molasses from the Doppler limit of a strong transition to below the recoil limit.

The second important parameter of the spectral profile is the relative frequency spacing between the sidebands. Figure 4 displays the improvement of the cooling result with decreasing spacing. The number of sidebands is varied while the velocity range covered remains fixed; the total power of all sidebands is kept constant. Independent of the detuning δ_c' (assuming that it is negative), we find a lower limit of Δp at a spacing $\omega_{sp} = \Gamma$. Sidebands with a spacing smaller than Γ seem to be inefficient, because the rate of nonresonant excitation of $p = 0$ atoms increases with decreasing spacing and leads to increased heating of the cold distribution.

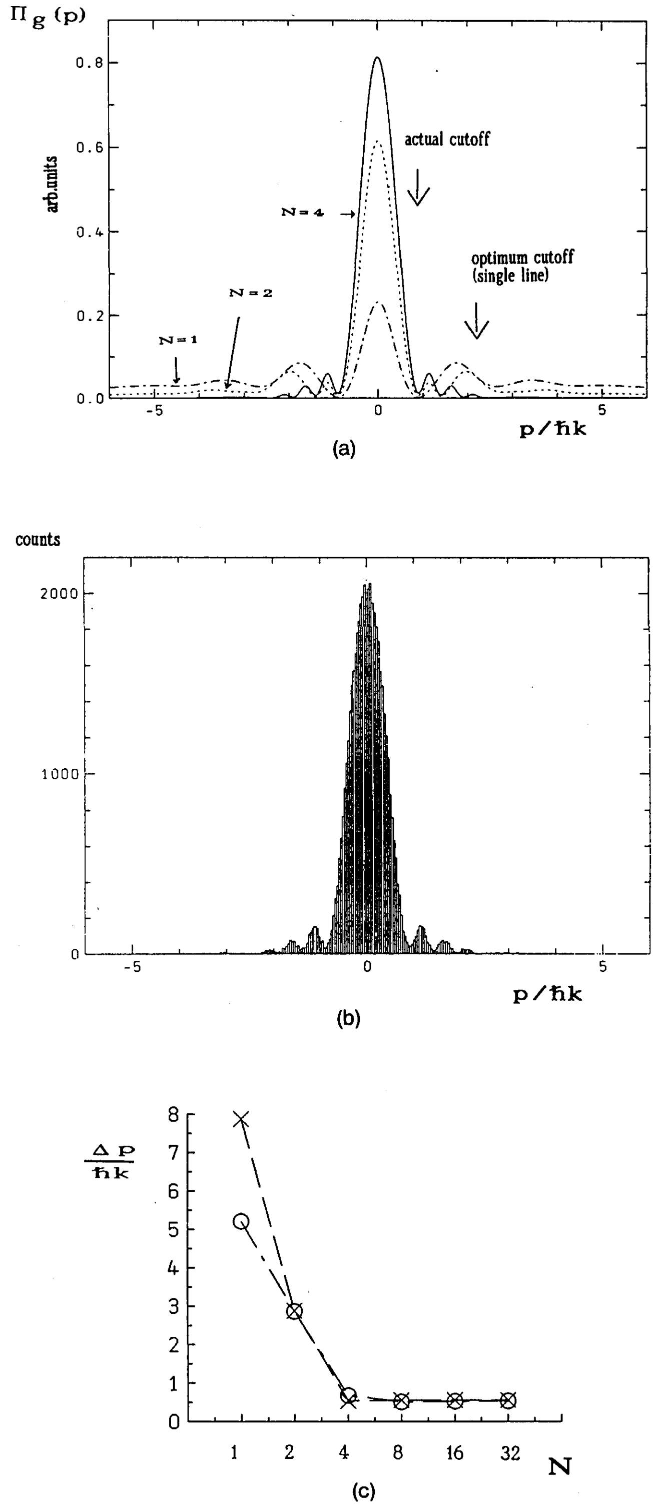


Fig. 3. Stationary momentum distributions for an increasing number of sidebands N. The spacing ω_{sp} is $0.5kv_R$, so that $(N-1)$ is proportional to the covered frequency range. (a) The numerical solutions of Eq. (7) for $N = 1, 2, 4$. The cutoff detuning is chosen to be $\delta' \approx -0.9\ kv_R$. A sideband spacing of $\omega_{sp} = 0.5kv_R$ provided a covered momentum range of $1.5\hbar k$ for the $N = 4$ sidebands. (b) Monte Carlo solution corresponding to (a) for $N = 4$, the ordinate: atoms per momentum channel $\hbar k/25$. (c) Δp as a function of the number of sidebands N; δ_c and ω_{sp} are as in (a) and (b), with $\Delta p = \Delta p_{rms}$ (crosses) and $\Delta p = \Delta p_{85\%}$ (circles).

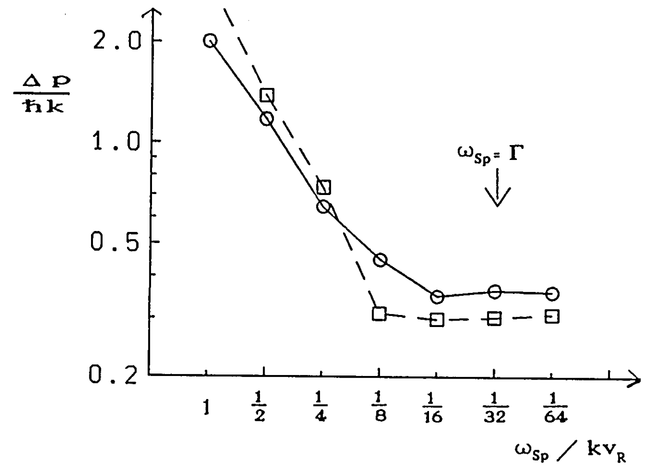


Fig. 4. Optimization of sideband frequency spacing: circles, $\Delta p_{\mathrm{rms}}/\hbar k$; squares, $p_{85\%}/\hbar k$. $v_R/v_D = 32$, the detuning $\delta_c' = -kv_R$, and the fixed frequency range covered by the lines is $2kv_R$. Note that the ordinate scale is logarithmic.

The last condition, $\omega_{\mathrm{sp}} = \Gamma$, leads us to an excitation profile that looks like a smoothed square profile with residual comb structure and a nonvanishing off-resonant excitation rate at $p \approx 0$. The ratio $\gamma_+(\hbar k)/\gamma_+(0)$ is independent of P_0 in the low-intensity limit and is typically 10^2 (e.g., when $kv_R = 30\Gamma$, other parameters optimized).

C. Quantum Limit

An ideal square excitation profile that totally excludes reexcitation of $p = 0$ atoms cannot be realized by any scattering force cooling scheme. In other words, an ultimately dark state of a V-level atom in the field of σ^+–σ^- light is not possible, and therefore in no way can the recoil heating be reduced to zero. This is the main discrepancy with the calculation in Ref. 7. [However, in the case of a Λ-level atom a superposition of momentum states allows a formation of an absolutely nondecaying (dark) state.[14,15]] The situation of a weakly decaying state resembles the situation of shelved ions[16]: the excitation profile provides a dark shelf in momentum space. On the other hand, the unavoidable residual recoil heating does not limit the minimum energy to the recoil energy. The postulation of the recoil limit[17] did not allow for an integral definition of the heating effect that involves π_g and γ_+ as weight functions strongly varying within the recoil width [see Eq. (4.5) of Ref. 4].

The result of the Monte Carlo calculations according to Eq. (7) shows that a compression down to the Doppler limit of the narrow transition will be possible, i.e., down to an uncertainty of the kinetic energy of $E_{\mathrm{kin}} \approx \hbar\Gamma$ [$\ll(\hbar k)^2/m$].

To find the quantum limit of broadband cooling we calculated stationary momentum distributions by using excitation profiles of the optimized form described above and by varying the cutoff detuning δ_c'. The compression of the momentum spread is improved by decreasing $|\delta_c'|$ but limited by the variation of the residual excitation rate, which varies approximately proportional to $1/|\delta_c'|$ (sum over Lorentzians).

We found a clear minimum of the momentum spread at an optimum detuning,

$$|\delta_c'(\mathrm{opt})| \approx 1.8(kv_R\Gamma)^{1/2},$$

if the sideband spacing equals Γ. This optimum refers to the smallest Δp as measured by the 85th percentile of the distribution [Fig. 5(a)]. The mean quadratic momentum spread is usually higher by a factor of 2. This is due to the special form of the distribution consisting of a pronounced peak and flat wings extending to approximately 2.5 $\hbar k$ and giving a (relatively) large contribution to the mean-square momentum. The narrower curve A of Fig. 5(b) has a *larger* quadratic momentum spread than the broader curve B.

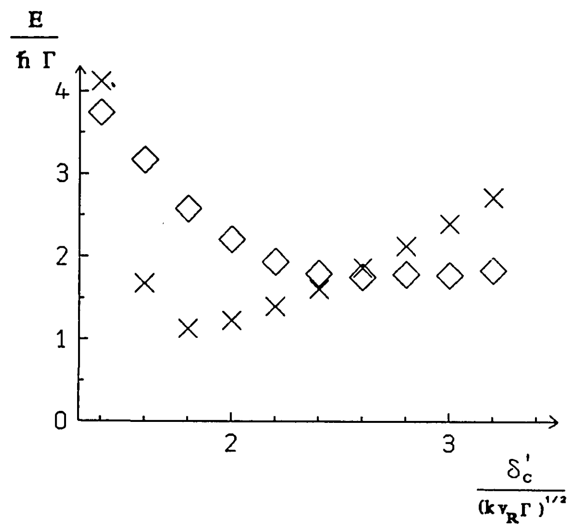
It is remarkable that for broadband cooling the optimum cutoff detuning is proportional to the square root of the linewidth,

$$|\delta_{\mathrm{opt}}| \approx (\Gamma k v_R)^{1/2},$$

and directly proportional to the expected velocity spread Δv,

$$|\delta_{\mathrm{opt}}/k| \approx (v_R v_D)^{1/2} = (\hbar\Gamma/m)^{1/2} \approx \Delta v,$$

where $v_D = \Gamma/k$. This should be compared with the case of monochromatic narrow-line cooling, where the optimum de-



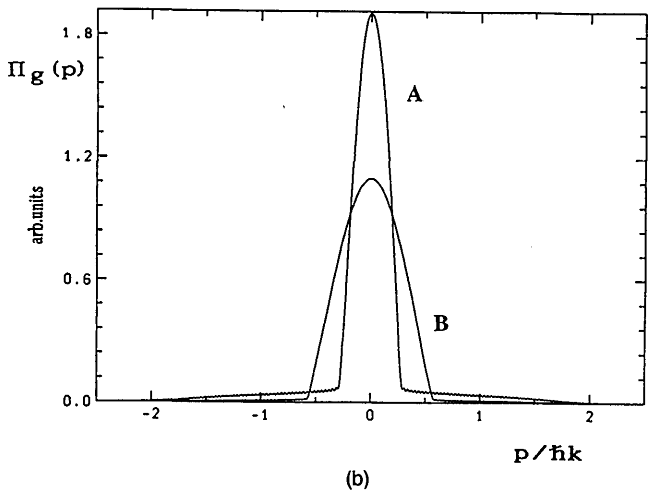


Fig. 5. (a) Kinetic energy versus detuning δ_c' for $v_R = 30v_D$. The ordinate: energy in units of $\hbar\Gamma$; the abscissa: detuning δ_c' in units of $(kv_R\Gamma)^{1/2}$. $\omega_{\mathrm{sp}} = \Gamma$, with $\Delta p_{\mathrm{rms}}/\hbar k$ (diamonds) and $\Delta p_{85\%}/\hbar k$ (crosses). (b) Momentum distributions $\delta_c' = -1.8(kv_R\Gamma)^{1/2}$ (curve A) and $\delta_c' = -3.6(kv_R\Gamma)^{1/2}$ (curve B). The normalization is as in Fig. 3(a).

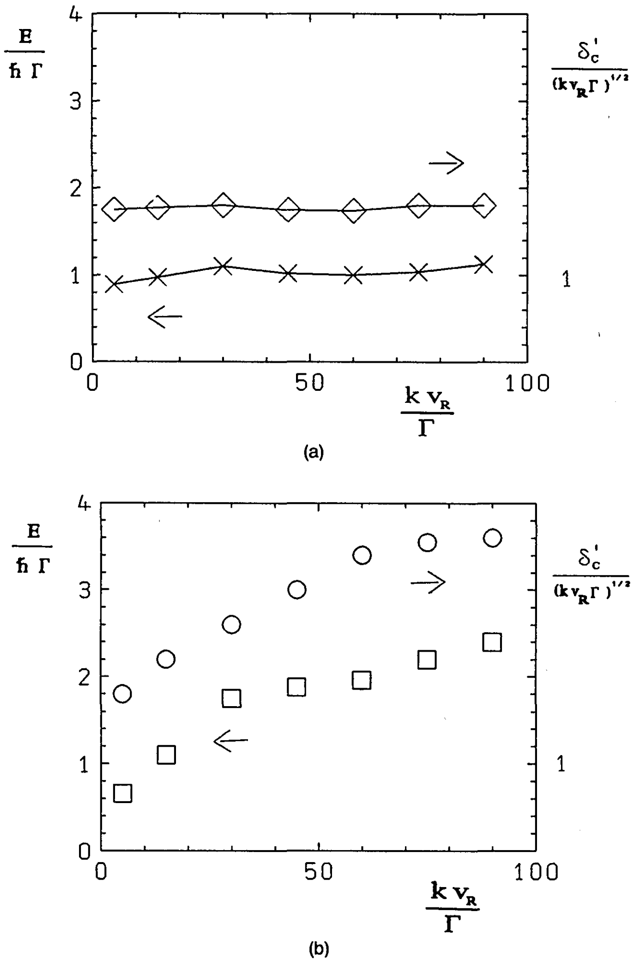


Fig. 6. Optimum detuning in units of $(kv_R\Gamma)^{1/2}$ and the minimum kinetic energy in units of $\hbar\Gamma$ as a function of kv_R/Γ calculated from (a) $\Delta p_{85\%}$ and (b) $\Delta p_{\rm rms}$.

tuning $-2.2kv_R$ and the residual momentum spread $\Delta p \approx 0.7\ \hbar k$ are independent of the linewidth, and with the usual case of a broad transition, where the optimum detuning is half the linewidth $\delta \approx -\Gamma/2$. This circumstance is related to the fact that there is no linear damping force but an approximate step-function dissipative force counteracted by residual heating for $p = 0$.

For large spacing $\omega_{\rm sp}$, i.e., an increasingly single-line nature of the spectral profile, the optimum δ_c increases and reaches the single-line value $\delta_c \approx -2kv_R$ for a large $\omega_{\rm sp}$. Using the optimum detuning for each linewidth Γ, we computed the minimum kinetic energies up to a ratio $kv_R \approx 100\Gamma$ (Fig. 6). For the Δp from the 85% width of the distribution, our results clearly reach the quantum limit $\hbar\Gamma$ independently of the linewidth [Fig. 6(a)]. The numerical results for the mean-square spread seem to indicate a weak dependence of the linewidth. However, it should be noted that the distributions are not Gaussian and do not show scale invariance. The leaking of probability into the wings (which cannot be totally suppressed) extends to a range $2\hbar k$ independently of the linewidth, so that their contribution to the mean-square momentum is not reduced so strongly as the 85% width of the distribution is reduced with decreasing linewidth.

It may be noted that the quantum limit $\hbar\Gamma$ corresponds to a velocity of $\approx$2 mm/sec for calcium, for example, and a temperature of 33 nK. For the region of broad lines ($kv_R/\gamma \to 0$) the quantum limit converges to the Doppler limit $\hbar\Gamma/4$.

D. Kinetic Limitations for the Cooling of Alkaline Earths

As stated in Section 1, the narrowness of the line results in an extremely slow evolution of the momentum distribution and makes it practically impossible to cool an initial ensemble with a velocity width larger than a few recoil velocities by a single narrow laser line. For a two-step cooling scheme for alkaline earths an initial ensemble may be chosen that has been cooled to the Doppler limit of the strong ($^1S_0$–$^1P_1$) transition with a linewidth $\Gamma_{\rm st}$. During the second cooling stage with the narrow transition the first cooling laser is switched off.

Let us first estimate the deceleration time for monochromatic cooling. For the corresponding initial momentum $p_1 \approx (\hbar\Gamma_{\rm st}/2m)^{1/2}$ the scattering rate would be negligible if the detuning were chosen near optimum ($\delta' = -2\hbar k^2/m$) for the slow transition. The time T required to reduce an initial momentum p down to zero in molasses has been derived in Ref. 4; for a detuning $2\hbar k^2/m$ it is

$$T \approx p^4/(8\hbar^2\Gamma m^2\Omega^2),$$

where Γ is the linewidth and Ω is the Rabi frequency of the narrow transition. Inserting p_i from above and $\Omega = \Gamma$, we have

$$T \approx (1/\Gamma)({\Gamma_{\rm st}}^2/\Gamma)^2. \tag{10}$$

The quotient of the broad linewidth over the narrow linewidth ($\Gamma_{\rm st}/\Gamma$) may easily exceed 10^5 (see Fig. 1), in which case cooling will not be observable.

A multiline scheme, on the contrary, is designed to provide a maximum deceleration rate $\hbar k/2\tau$ during the whole cooling process, from the molasses momentum of the first strong transition down to the recoil momentum of the second slow transition. In this case the required time would be

$$T \approx 2\tau(\hbar\Gamma_{\rm st}m/2)^{1/2}/\hbar k. \tag{11}$$

The corresponding parameters for the alkaline earths calcium, magnesium, and strontium are listed in Table 1.

This estimate does not include the time of the final stage of cooling from the recoil momentum down to the quantum limit momentum. For linewidths down to $kv_R/100$ we checked within our Monte Carlo calculations that the average number of photons scattered in this stage is of the order of the reciprocal probability to hit the interval $(2m\hbar\Gamma)^{1/2}$ within an interval $\hbar k$. Nevertheless, because of the magnitude of τ, even interaction times of 50τ may cause experimental problems. For a maximum deceleration $\hbar k/2\tau$, atoms diffuse over a free-flight distance

$$l \approx \tau(p_i)^2/(4m\hbar k) = \tau(\hbar\Gamma_{\rm st}/8\hbar k) \tag{12}$$

during the time of deceleration down to the molasses limit. To estimate the total spread of an ensemble with an initial momentum spread p_i as assumed above, we performed a Monte Carlo simulation for the atomic motion in momentum and position space. Of course, Heisenberg's uncertainty relation limits the resolution of the position. Since the momentum in our treatment is required to be defined better than $\Delta p \approx m\Gamma/k$, we have to obey $\Delta x \approx \hbar/\Delta p \approx v_R\tau$. For the slow transitions under consideration, $\Delta x \approx v_R\tau$ is in the

Table 1. Broadband Cooling Parameters for Alkaline Earths[a]

Parameters	Magnesium	Calcium	Strontium
λ (nm)	457	657	689
τ (sec)	4.6×10^{-3}	0.5×10^{-3}	21×10^{-6}
$\hbar k/m$ (cm/sec)	3.64	1.52	0.66
$\hbar k^2/m\Gamma$	2.3×10^3	73	1.26
Γ_{st}/Γ	2.3×10^6	1.1×10^5	4.2×10^3
$[(m\hbar\Gamma_{st}/2)^{1/2}]/\hbar k$	22.2	27.3	40.15
T (msec)	204	27.4	1.7
l (mm)	234	2.83	0.2
$2\hbar\Gamma/k_B$ (nK)	3.3	30	727

[a] The wavelength λ, the lifetime $\tau = 1/\Gamma$, the recoil velocity $\hbar k/m$, and the ratio of the recoil shift to the linewidth refer to the $(^1S_0-^3P_1)$ intercombination line. Γ_{st} is the linewidth of the strong $(^1S_0-^1P_1)$ cooling transition. The cooling time T is set equal to $2\tau\Delta p_i/\hbar k$; the initial momentum spread Δp_i is taken as the molasses momentum corresponding to the strong transition. The typical stopping distance is of the order of the listed parameter $l = \Delta p_i T/2m$. The value of the limit temperature is calculated according to $E_{kin} = \hbar\Gamma$.

millimeter range, which is much larger than the optical wavelength λ.

The results of the Monte Carlo simulations agree in principle with relations (11) and (12). The calculated compression of the whole distribution function down to the range of the recoil momentum takes approximately twice the time as estimated in relation (11), owing to the wings of the initial Gaussian distribution. During this time the corresponding expansion is approximately four times the stopping distance given in relation (12).

The comparison of the parameters of the alkaline earths shows that calcium will be an interesting candidate for this cooling scheme, because on one hand the reduction of the temperature is larger than for strontium, while on the other hand the spatial extension and the cooling time are experimentally feasible.

4. CONCLUSION

We have presented a numerical investigation of the limit of laser cooling on a narrow atomic transition with broadband excitation. The probability transfer in momentum space was described by an integral equation whose solutions were obtained by numerical and Monte Carlo methods. The achievable residual kinetic energies are of the order of the width $\hbar\Gamma$ of the excited state if a detuning $\delta_c' \approx -2(kv_R\Gamma)^{1/2}$ proportional to the square root of the linewidth is chosen.

The application of broadband laser light appears to be the only feasible way to cool an atomic ensemble with a Doppler width much larger than the recoil width using a narrow transition. For the case of the intercombination line of alkaline-earth elements, cooling in a finite interaction time and volume was discussed. A proper candidate for experimental realization of such a scheme seems to be calcium.

Multiline schemes open unique possibilities for the cooling on narrow lines because the large velocity acceptance is combined with a narrow final velocity distribution. Such schemes can also be combined with other laser cooling and trapping schemes and will improve them significantly.

The experimental realization profits from the discreteness of the laser lines because the slope of the excitation profile is still determined by the atomic Lorentzian if it is possible to suppress the intensity envelope of the sidebands within one frequency interval ω_{sp}. For the carrier, dye lasers with subkilohertz stability, as recently demonstrated,[18] will be required.

APPENDIX A

Solving the subset of the density-matrix Eqs. (3) that are diagonal with respect to the momentum for $\pi_+(p)$ as a linear function of $\pi(p) = \pi_g(p) + \pi^+(p) + \pi_-(p)$, we obtain

$$\frac{\pi_+(p)}{\pi(p)} = \frac{(\Omega^2/4)(A_-A_0 + \Omega^4/4)(A_0 + \Omega^2)}{A_+A_-A_0^2 + (A_+ + A_-)A_0B + C},$$

with

$$\delta_+ = \delta' - kp/m,$$

$$\delta_- = \delta' + kp/m,$$

$$\delta_0 = \delta_+ - \delta_- = -2kp/m,$$

$$A_+ = \Gamma^2/4 + \delta_+^2 + \Omega^2/2,$$

$$A_- = \Gamma^2/4 + \delta_-^2 + \Omega^2/2,$$

$$A_0 = \Gamma^2/4 + \delta_0^2,$$

$$B = (\Omega^2/2)(5\Omega^2/4 + 3\Gamma^2/4 + \delta_+\delta_-),$$

$$C = (\Omega^6/4)(\Omega^2 + 3\Gamma^2/4 + \delta_+\delta_-).$$

This reduces to lowest order in Ω^2:

$$\frac{\pi_+(p)}{\pi(p)} = \frac{(\Omega^2/4)}{A_+} \approx \frac{\pi_+(p)}{\pi_g(p)}.$$

ACKNOWLEDGMENTS

The authors thank J. Dalibard and A. Aspect for fruitful discussions.

Financial support by the Deutsche Forschungsgemeinschaft (Er-113/4-2) is gratefully acknowledged.

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