Quantum Electronics Letters

Linewidth Determination from Self-Heterodyne Measurements with Subcoherence Delay Times

L. E. RICHTER, H. I. MANDELBERG, M. S. KRUGER, AND P. A. McGRATH

Abstract—**The behavior of the power spectrum of an external cavity semiconductor laser has been studied using a delayed self-heterodyne interferometric technique that uses delay times less than the laser's coherence time. Experimental results show that the resulting power spectrum is consistent with the theoretical model. However, there is evidence that additional frequency fluctuations are present that cause the delta function portion of the power spectrum to have a finite width.**

WITH the current interest in coherent systems for optical communications, much effort is being directed toward the development of narrow linewidth semiconductor lasers. By using external cavity configurations, linewidths of less than 20 kHz have been reported [1], [2]. Accurate measurements of these narrowed spectral linewidths are required to characterize system performance. Several methods have been used for diode laser linewidth determination [1]–[9]. The most common method, which uses a Fabry–Perot interferometer, has insufficient resolution to measure these narrowed linewidths. The delayed self-heterodyne technique first proposed by Okoshi *et al.* [9] offers the highest resolution, but requires that the delay time be much longer than the laser's coherence time.

In this letter, we report the experimental results of a high-resolution linewidth measurement (full width at half maximum) which uses delay times significantly less than the laser's coherence time. The modified Mach–Zehnder interferometer is illustrated in Fig. 1. The external cavity laser was composed of an RCA CDH-LOC diode laser ($\lambda = 0.82$ μm), a collimating objective, an 80 percent reflective output coupler, and a 0.25 mm etalon. The front facet of the diode was anti-reflection coated and the rear facet was coated for high reflectivity. This external cavity configuration produced a narrow-line source with which we could evaluate this interferometric technique. Calculations from theory [5] predict that this particular external cavity laser should have a subkilohertz linewidth.

The output from this laser was directed through an optical isolator into the interferometer. Immediately following the isolator, the beam was split into two paths by a 50/50 beamsplitter. A portion of one beam was constantly monitored with a scanning Fabry–Perot interferometer to ensure that the laser was operating in a single, stable longitudinal mode. The remainder of the beam was delayed by interchangeable lengths of single-mode optical fiber. The other beam was directed through an acoustooptic modulator operating at 54 MHz. The first-order deflected beam was focused into a short length (1 m) of fiber to ensure that the phase fronts of the two recombined beams would be well matched at the detector. The two beams were then recombined by a beamsplitter and directed into a photomultiplier tube. The resulting beat signal was displayed on an HP8568A spectrum analyzer and stored on an HP9826 computer.

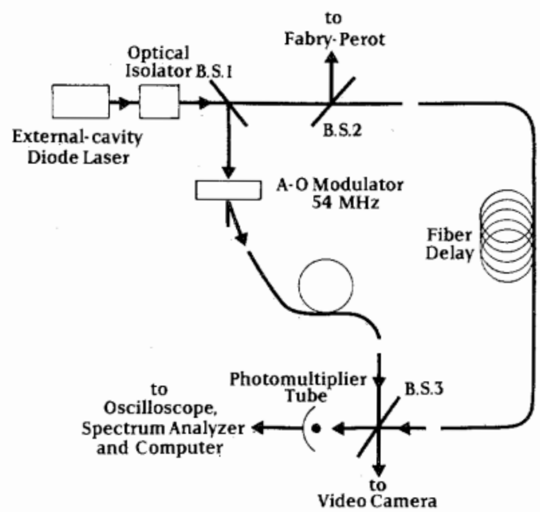


Fig. 1. Experimental arrangement.

Briefly reviewing the interferometric theory [10], [11], the total electric field incident on the detector $E_d(t)$ is found by summing the electric field from each arm of the interferometer:

$$E_d(t) = \sqrt{P} \cos(\omega_0 t + \phi(t)) + \sqrt{P_0} \cos((\omega_0 + \Omega) \cdot (t - \tau) + \phi(t - \tau)) \tag{1}$$

where P_0 is the laser's output power, ω_0 is the laser frequency, τ is the time delay of one path with respect to the other path, and Ω is the offset frequency. The total detected intensity is then given by

Manuscript received November 8, 1985; revised June 20, 1986. The work of L. E. Richter was submitted to the University of Maryland, College Park, in partial fulfillment of the requirements for the Master of Science degree.

The authors are with the Laboratory for Physical Sciences, 4928 College Ave., College Park, MD, 20740.

IEEE Log Number 8610501.

0018-9197/86/1100-2070$01.00

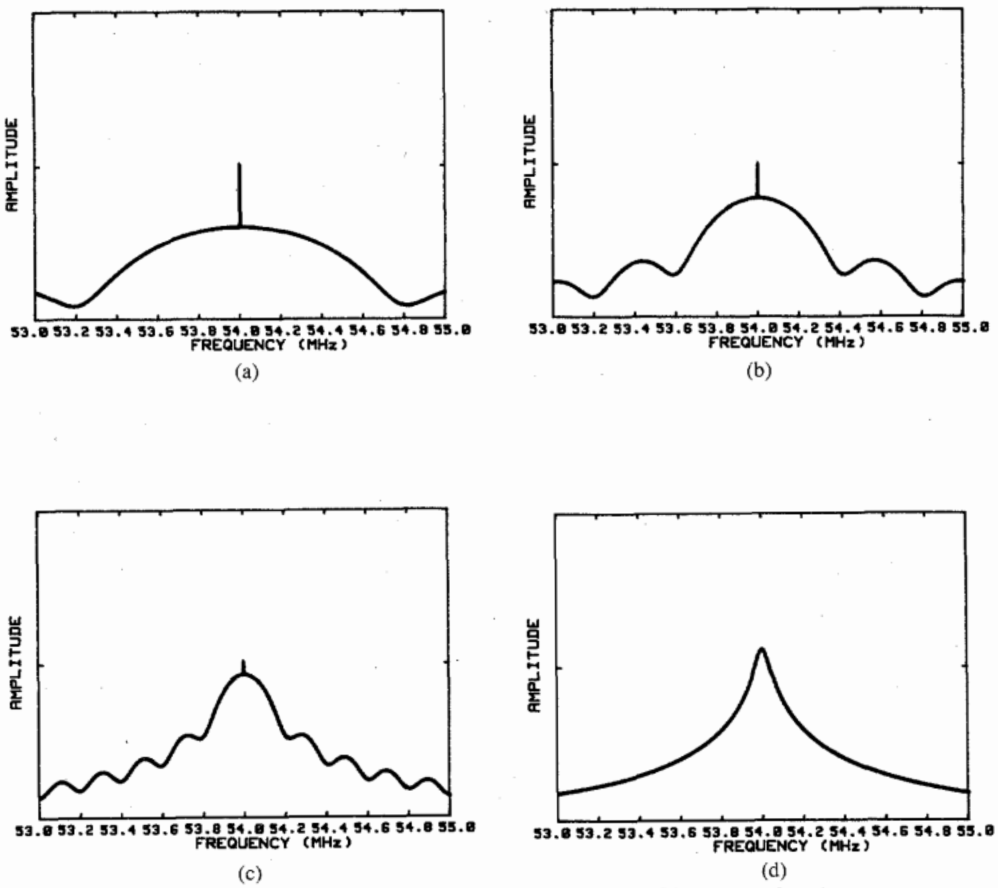


Fig. 2. Computer simulation of the power spectrum $S(\omega, \tau)$ as a function of $|\tau|/\tau_c$ where (a) $|\tau|/\tau_c = 1.25$, (b) $|\tau|/\tau_c = 2.5$, (c) $|\tau|/\tau_c = 5.0$, (d) $|\tau|/\tau_c = 30.0$. Arbitrary logarithmic amplitude.

$$I_d(t) = P_0 + P_0 \cos\left[-\Omega t + (\omega_0 + \Omega)\tau + \phi(t) - \phi(t - \tau)\right]. \tag{2}$$

The resulting autocorrelation function $\gamma(\delta\tau, \tau)$ is then found to be

$$\gamma(\delta\tau, \tau) = \tfrac{1}{2}P_0^2 \cos(\Omega\delta\tau)\, \langle \cos(\Delta\phi(\delta\tau, 0) - \Delta\phi(\delta\tau - \tau, \tau))\rangle \tag{3}$$

where $\Delta\phi(t_1, t_2) = \phi(t' + t_1) - \phi(t' + t_2)$, the phase difference between times t_1 and t_2. Since the phase jitter is assumed to have a Gaussian distribution, the relationship $\langle \cos x\rangle = e^{-\langle x^2\rangle/2}$ can be used to simplify the expression for $\gamma(\delta\tau, \tau)$. In addition, the phase correlation of the recombined beams can be written as

$$\langle[\phi(t) - \phi(t - \tau)]^2\rangle = \frac{|\tau|}{\tau_c} \tag{4}$$

where τ_c is the coherence time. Then, with further manipulation, the autocorrelation function may ultimately be expressed as

$$\gamma(\delta\tau, \tau) = \frac{1}{2} P_0^2 e^{-|\tau|/\tau_c} \cos(\Omega\delta\tau) \cdot \exp\left[-\frac{1}{\tau_c}(\delta\tau - |\tau|)\right]. \tag{5}$$

Using the Wiener–Khintchine theorem and performing the integration, we arrive at the following expression for power spectral density:

$$S(\omega, \tau) = \frac{\frac{1}{2}P_0^2\tau_c}{1 + (\omega \pm \Omega)^2\tau_c^2}\left\{1 - e^{-|\tau|/\tau_c} \cdot \left[\cos(\omega \pm \Omega)|\tau| + \frac{\sin(\omega \pm \Omega)|\tau|}{(\omega \pm \Omega)\tau_c}\right]\right\} + \frac{1}{2}P_0^2\pi e^{-|\tau|/\tau_c}\,\delta(\omega \pm \Omega). \tag{6}$$

This equation indicates that as the delay time increases, the signal strength shifts from the delta function peak to the modified Lorentzian pedestal until the power spectrum becomes strictly Lorentzian. This true Lorentzian corresponds to the delay time when the phases of the optical field have become totally decorrelated. In addition, as the delay time increases, oscillations appear in the wings of the power spectrum which are due to the exponential portion of the power spectrum. This behavior is illustrated in Fig. 2. Our computations have shown that in order to measure the width of the pedestal portion of the beat signal, the delay time must be approximately six times the

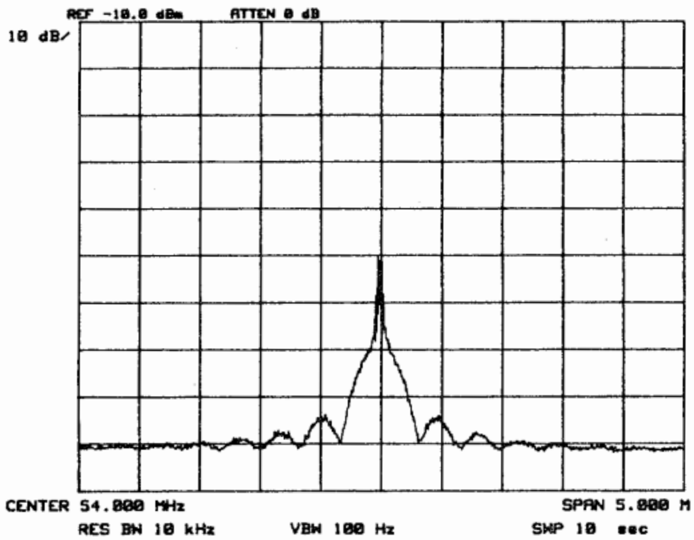


Fig. 3. Power spectrum observed for a delay time of 3 μs.

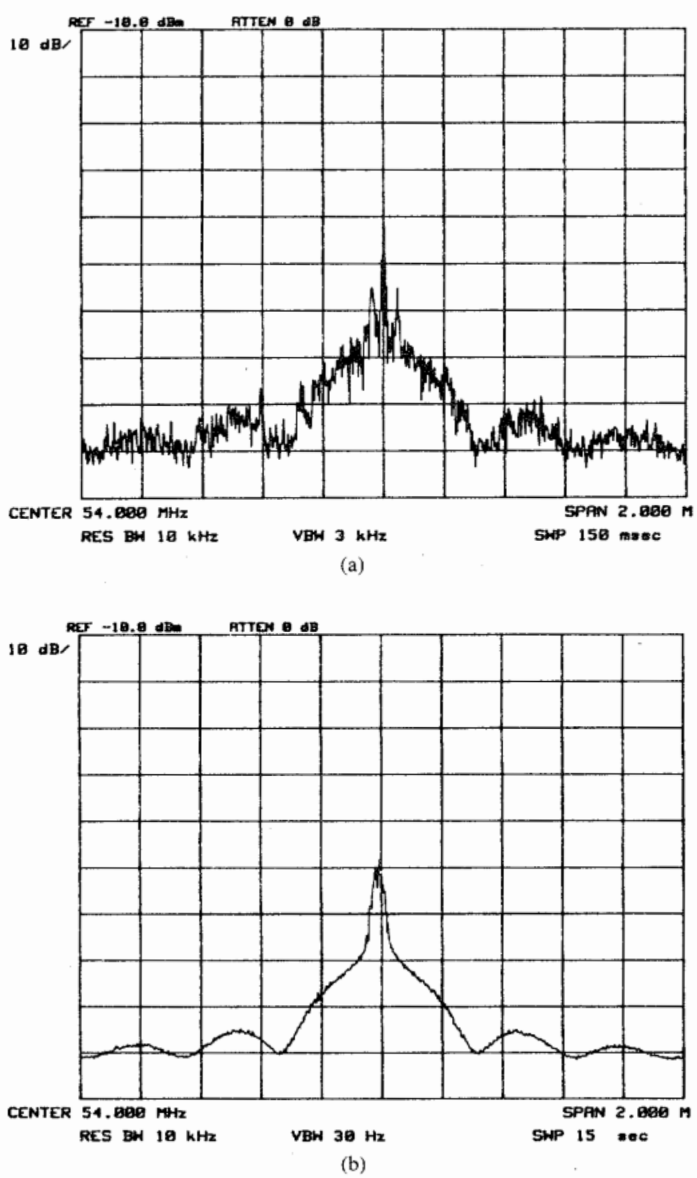


Fig. 4. Power spectrum observed for a delay of 3 μs; (a) 150 ms sweep, (b) 15 s sweep.

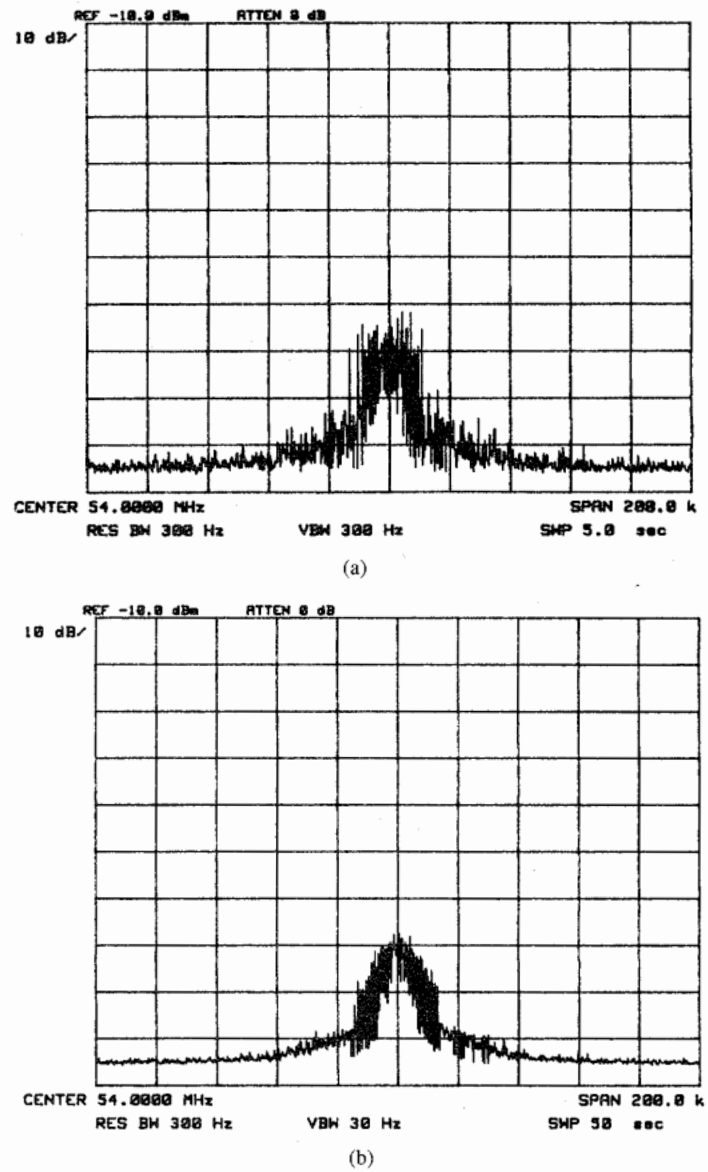
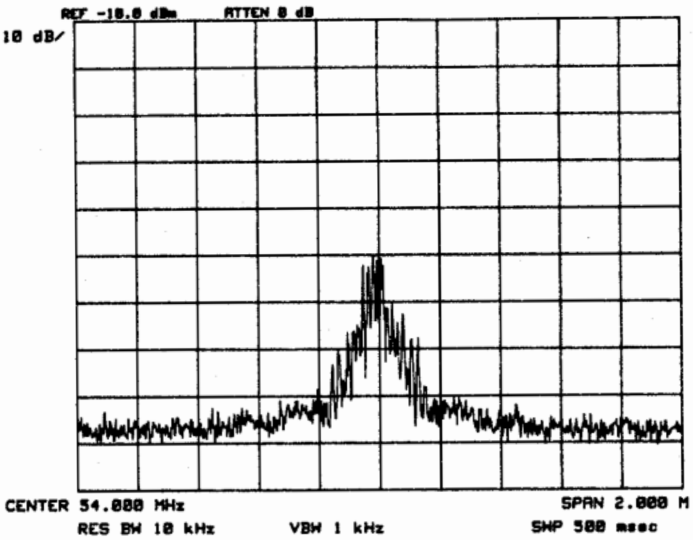


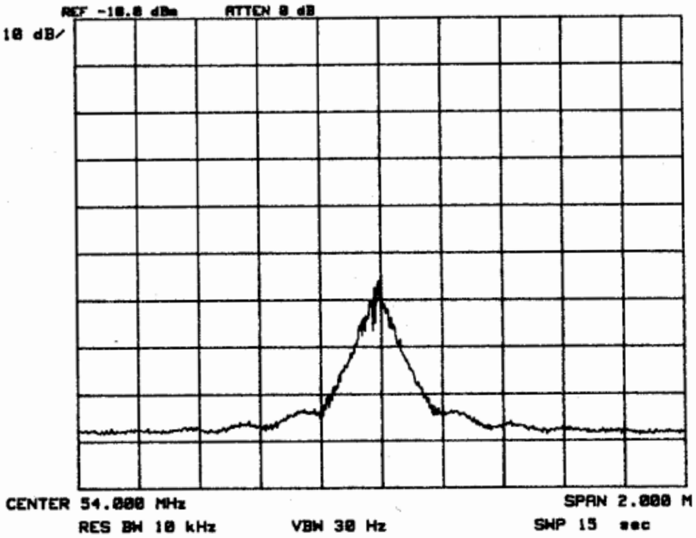
Fig. 5. Power spectrum observed for a delay time of 1.5 μs; (a) 5 s sweep, (b) 50 s sweep.

laser's coherence time. This implies that a fiber optic delay of more than 36 km would be required to enable the true measurement of a 10 kHz linewidth.

The power spectrum was observed for six different lengths of fiber; 1 m, 300 m, 600 m, 1.1 km, 1.7 km, and 2.8 km. These lengths correspond to delay times of approximately 0, 1.5, 3.0, 5.6, 8.5, and 14 μs. All measurements were taken with the laser operating at the same average output power level. The observed spectra appeared to follow the theoretically predicted behavior, with the sideband ripples forming as τ was increased toward τ_c and then disappearing as τ exceeded τ_c. A representative spectrum analyzer trace for $\tau = 3.0$ μs is shown in Fig. 3. The delta function portion of this relatively slow (10 s) trace was seen to have a finite width that was not spectrum analyzer resolution bandwidth limited. Additional spectrum analyzer traces for $\tau = 3$ μs are shown in Fig. 4 with different frequency scales and sweep speeds. Similar



(a)



(b)

Fig. 6. Power spectrum observed for a delay time of 5.6 μs; (a) 500 ms sweep, (b) 15 s sweep.

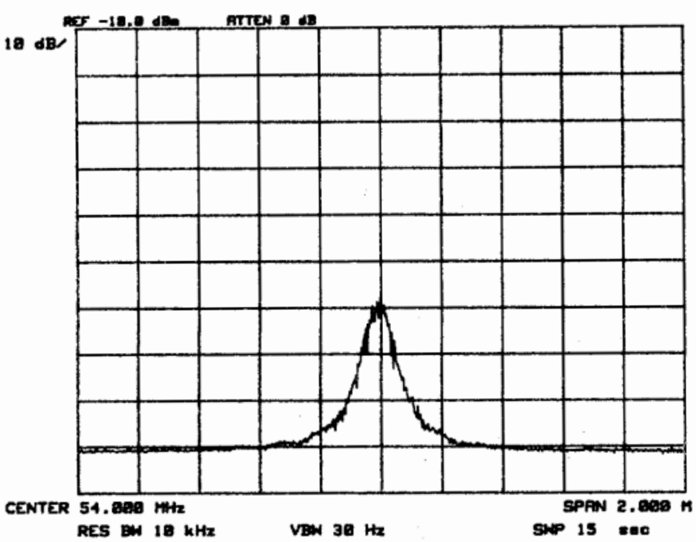


Fig. 7. Power spectrum observed for a delay time of 8.5 μs, 15 s sweep only.

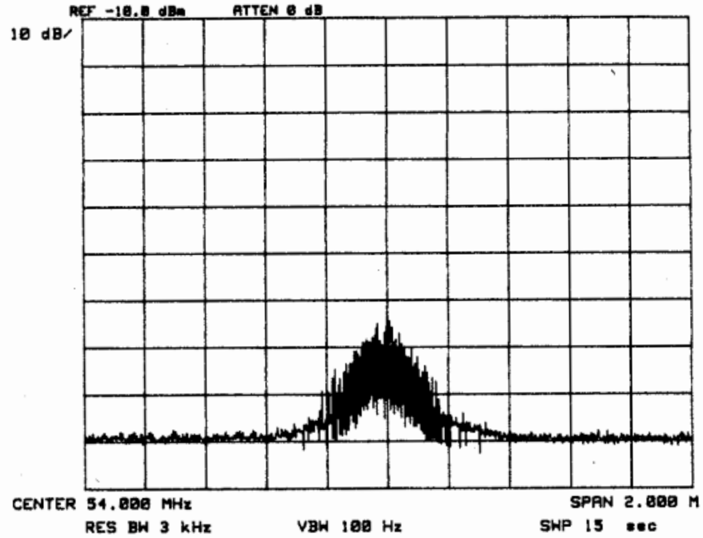


Fig. 8. Power spectrum observed for a delay time of 14 μs, 15 s sweep only.

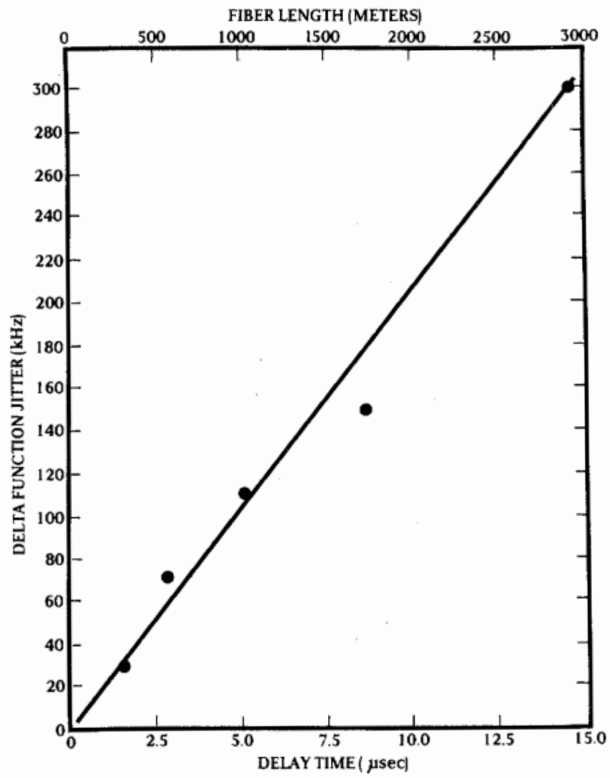


Fig. 9. Behavior of the delta function jitter width as a function of delay time (fiber length).

traces for the other delay times are shown in Figs. 5–8. The full width at half maximum of the delta function was observed to increase linearly with delay time as plotted in Fig. 9. This linearity implies the presence of jitter of the laser's center frequency whose periodicity is greater than the longest delay time used.

Analysis of the resulting data was performed using a weighted least squares approximation technique. This analysis provided an estimation of the laser's coherence time from the pedestal portion of the power spectrum. However, because the delta function jitter obscured the

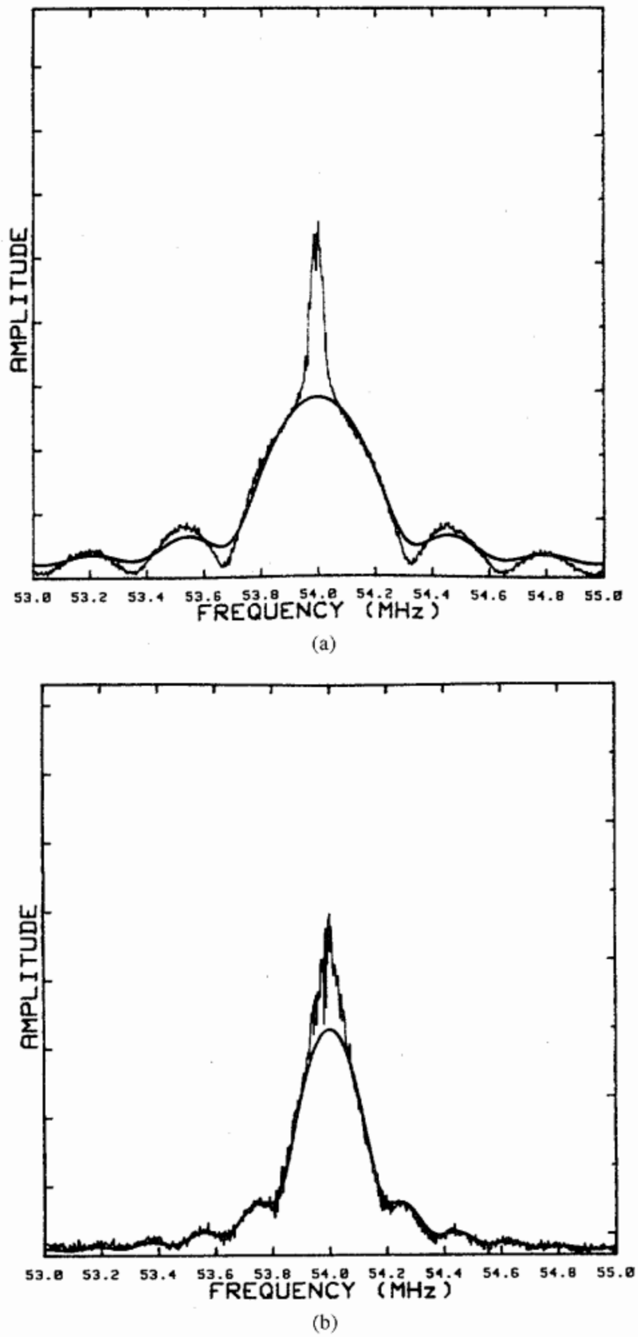


Fig. 10. Results of the least square approximation to the data. (a) Fit of data in Fig. 4(b), $\tau_{c\,\mathrm{fit}} = 4.1\ \mu s$. (b) Fit of data in Fig. 6(b), $\tau_{c\,\mathrm{fit}} = 6.5\ \mu s$.

pedestal for the longer delay times, results are presented only for the 3 and 5.6 μs delays. Since the jitter of the delta function precluded accurate estimation of the division of power between the pedestal and the delta function portions of the power spectra, no delta function is represented on the plots of the estimated power spectra. Representative estimates for these data are shown in Fig. 10(a) and (b), respectively. These figures indicate that the analysis fit the data reasonably well, yielding estimates for the coherence time ranging from 3 to 10 μs. However, the theory does not appear to adequately predict the depth of the sideband ripples. The broad range of estimated values of the coherence time can be attributed to the inherent mechanical and thermal instability of our external cavity laser.

In conclusion, the linewidth of a line-narrowed external cavity diode laser was measured using a self-heterodyne interferometric technique with multiple delay times less than the laser's coherence time. The data show the presence of low-frequency fluctuations that cause the delta function portion of the power spectrum to have a finite width. We believe that this technique may provide a viable way of measuring the linewidth of a very stable line-narrowed diode laser; however, in our laboratory setup, the results leave some uncertainty as to the laser's ultimate linewidth since the external cavity was quite unstable.

Acknowledgment

The authors would like to thank D. Rush, Dr. C. Davis, and Dr. P. T. Ho for their assistance throughout this work.

References

[1] R. Wyatt and W. J. Devlin, "10 kHz linewidth 1.5 μm InGaAsP external cavity laser with 55 nm tuning range," *Electron. Lett.*, vol. 19, pp. 110–112, 1983.

[2] C. Kuo and J. Van Der Ziel, "Linewidth reduction of 1.5 μm grating loaded external cavity and semiconductor lasers," in *Tech. Dig., Conf. Optical Fiber Commun.*, Feb. 1986, paper MI-3.

[3] M. W. Fleming and A. Mooradian, "Spectral characteristics of external cavity controlled semiconductor lasers," *IEEE J. Quantum Electron.*, vol. QE-17, pp. 44–59, Jan. 1982.

[4] A. R. Reisinger, C. D. David, Jr., K. L. Lawly, and A. Yariv, "Coherence of a room temperature CW GaAs/GaAlAs injection laser," *IEEE J. Quantum Electron.*, vol. QE-15, pp. 1382–1387, 1979.

[5] E. Patzak, A. Sugimura, S. Saito, T. Mukai, and H. Olesen, "Semiconductor laser linewidth in optical feedback configurations," *Electron. Lett.*, vol. 19, pp. 1026–1027, 1983.

[6] B. Daino, P. Spano, M. Tamburrini, and S. Piazzolla, "Phase noise and spectral lineshape in semiconductor lasers," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 266–270, 1983.

[7] Y. Yamamoto, T. Mukai, and S. Saito, "Quantum phase noise and linewidth of a semiconductor laser," *Electron. Lett.*, vol. 17, pp. 327–329, 1981.

[8] S. Machida, A. Kawang, K. Ishida, and H. Tsuchiya, "Interference of an AlGaAs laser diode using 4.15 km single mode fiber cable," *IEEE J. Quantum Electron.*, vol. QE-15, pp. 535–537, 1979.

[9] T. Okoshi, K. Kikuchi, and A. Nakayama, "Novel method for high resolution measurement of laser output spectrum," *Electron Lett.*, vol. 16, pp. 630–631, 1980.

[10] F. B. Gallion and G. DeBarge, "Quantum phase noise and field correlation in single frequency semiconductor laser systems," *IEEE J. Quantum Electron.*, vol. QE-20, pp. 343–349, 1984.

[11] K. Peterman and E. Weidel, "Semiconductor laser noise in an interferometer system," *IEEE J. Quantum Electron.*, vol. QE-17, pp. 1251–1256, 1981.